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Analysis of Faculty Teaching based on Student feedback using Fuzzy Relation Equation

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Abstract

Nowadays many institutions have started paper based approach or web-based approach to gather student feedback on faculty teaching. In recent years, many researches have confirmed what most teachers already knew: providing students with meaningful feedback can greatly enhance their performance day by day. Students have very much concerned about their teachers who have played a vital role on their life in both outside or inside of the institutions which helps them to justify a teacher and is reflected in the feedback form. In this paper, we have approached a ranking method of the given feedback's factors of teacher's performances with the help of various fuzzy operators which is used to compute the solution of Fuzzy Relation Equations (FREs) and defuzzify the obtained results to rank performances. It is fits to use fuzzy theory in general and FREs in particular to accomplish this aim; linguistic variables are introduced to deal with uncertainty factors. FREs are the best suited tool when the data is an unsupervised one.

Keywords: *Fuzzy relation equations, Fuzzy operators, Alpha-composition, Feedback*

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1. Introduction

A fuzzy model is a finite set of fuzzy relations that form an algorithm for determining the outputs of a process from some finite number of past inputs and outputs. A fuzzy model can be used in applied mathematics to study social and psychological problem and also used by doctors, engineers, scientists, industrialists and statisticians. The notion of fuzzy relation equation based upon the max-min composition was first investigated by Sanchez [15]. He studied conditions and theoretical methods to resolve fuzzy relations on fuzzy sets defined as mappings from the sets to the interval $[0,1]$. Some theorems for existence and determination of solution of certain basic fuzzy relation equations were given by him. However the solution obtained by him is only the greatest element (or the maximal solution) derived from the max-min (or min-max) composition of fuzzy relation. The max-min composition is commonly used when a system requires conservative solutions in the sense that the goodness of one value cannot compensate the badness of another value. In reality there are situations that allow compensatability among the values of a solution vector. In such cases the in operator is not the best choice for the intersection of fuzzy sets, but max-product composition, is preferred since it can yield better or at least equivalent result. Many researchers have made research for finding the best solutions of Fuzzy Relation Equations (FREs). The researchers found in some cases that the minimum solution of FREs didn't exist, was not unique or there was no solution. By observing the nature of such solution set, researchers yield some methods somehow analytical and numerical to find the optimal solution e.g. Di Nola Sessa, Pedrycz in 1989 [6], Ciaramella et al. 2006 [4] have made a great contributions to model the FREs numerically with neural network and then adjust the problem according to the working algorithm. Many algorithms are developed by the use of FREs in order to solve the optimization problems. Among them Branch Point (BP) algorithm [11], Banach bound algorithm and genetic algorithms are frequently used techniques to investigate the solution of optimization problems with the help of FREs. In 1979. E Sanchez in his paper on Resolution of composite FREs [14] provided a method for the solution of FREs and proposed an algorithm calculating the supreme value and also the interval of solutions that provides exactly all the widest solution set which meet the goals. In 2000 Bourke and Grant Fisher [2], [3] analyzed optimizing algorithms for authenticity of the relation matrix and summarized them. they focused on neuro-based approach to find optimal solution of FREs. In 2000 Tatiana Kiseliova [9] developed a theoretical comparison of disco and cadiag-II like systems for medical diagnoses using fuzzy approach. These systems are characterized by a fuzzy relation based scenario and compared with cadiad II like systems based on fuzzy technologies. In 2009, Martina Stepnica, Bernard De Baets, Lenka Noskov [10] proposed an arithmetic fuzzy model in which they used some other fuzzy relations very closely to Takagi-Sugeno models under some linear requirements. S. Jain and K. Lachhwani [8] proposed a methodology for the solution of multi objective programming problem in FREs to find out the optimal solution among the bunch of solution of the given problem when the factors are in linguistic forms.

In recent years, a feedback system of a faculty in an institute is very much effective to improve teacher effectiveness, maximize their skill set with an effective classroom disciplinary climate and outside of the classroom also. In the feedback system, the data are in linguistic form. In this paper we have used FREs to represent the feedback system and use some operators which operate on FREs to find out the optimal solution among the bunch of solution of the given problem and ranking the performances of the faculty. The paper is organized in five sections. Section two presents the basic definition of Fuzzy Relation Equations (FREs). Section three describes various fuzzy operators and their operations on FREs. Later gives some lemmas and theorems on those operators and some necessary condition for the existence of those equations. Section four gives the various mathematical approaches of fuzzy operators on our discussed problems. Finally, Section five draws conclusions based on our study.

2. Preliminaries

Since the introduction of fuzzy set theory by Zadeh [16] in 1965 a significant group of papers has appeared, devoted to theoretical and application aspects of fuzzy relational equation studied by Sanchez [15] and forming a generalization of well known Boolean equations. FREs are associated to the composition of fuzzy relations and have been used to investigate applications like approximate reasoning, time series forecast, decision making, fuzzy control, as an appropriate tool for handling and modeling of non-probabilistic from of uncertainty etc. Many papers have investigated the capacity to solve FREs (in) [1, 5, 6, 12, 13]. Before discussing the algorithm of FREs we shall give the definition of fuzzy relations.

2.1 Fuzzy Relations

Let A and B be two crisp sets. Then a fuzzy relation from A to B is a fuzzy set having degree of membership whose value lies in [0,1], $\mu_R: A \times B \rightarrow [0,1]$;

$R = \{((x,y)) | \mu_R(x,y) \geq 0, x \in A, y \in B\}$

The domain of this relation is defined as $\mu_{dom(R)}(x) = \max_{y \in B} \mu_R(x,y)$ for each $x \in A$. The range of $R(A,B)$ is a fuzzy relation on B, now the membership function is defined by $\mu_{ran(R)}(x) = \max_{x \in A} \mu_R(x,y)$ for each $y \in B$. That is the strength of strongest relation that each element of B has to an element A is equal to the degree of that elements membership in the range of R. A fuzzy relation $R(X,X)$ is reflexive if and only if $R(x,x) = 1\ \forall x \in X$, if this is not the case for some $x \in X$, the relation is called irreflexive, if it is not satisfied $x \in X$, the relation is called antireflexive. A weaker form of reflexivity referred to $x \in X$ reflexivity denoted by $R(x,x) \geq \in$ where $0 < \in < 1$. A fuzzy relation is called symmetric if and only if $R(x,y) = R(y,x) \forall x,y \in X$. If this relation is not true for some $x,y \in X$, the relation is anti-symmetric. Furthermore when $R(x,y) > 0$ and $R(y,x) > 0$ implies $x = y\ \forall\ x,y \in X$, the relation R is called anti-symmetric. A fuzzy relation $R(X,X)$ if $R(x,z) \geq \max - \min_{y \in Y}[R(x,y), R(y,z)]$ is satisfied for each pair $(x,z) \in X^2$. A relation failing to satisfy for some members of X is called non-transitive and if $R(x,z) > \max - \min_{y \in Y}[R(x,y), R(y,z)] \forall (x,z) \in X^2$. Then the relation is called anti-transitive.

2.2 Fuzzy Relation Equations

The notion of fuzzy equation is associated with the concept of composition of binary relation. The composition of two fuzzy relation $P(X,Y)$ and $Q(Y,Z)$ can be defined in general in terms of an operation on the membership matrices P and Q that resembles matrix multiplication. This operation involves exactly the same combinations of matrix entries as in the regular matrix multiplication. However the multiplication and addition that are applied to these combinations in the matrix multiplication are replaced with other operations, these alternative operations of fuzzy set intersection and union respectively. In the max-min composition the multiplication and addition are replaced by the min and max operation. Consider three Fuzzy binary relations $P(X,Y), Q(Y,Z)$ and $R(X,Z)$ which are defined on the sets $X = \{x_i | i \in I\}$, $Y = \{y_j | j \in J\}$ and $Z = \{z_k | k \in K\}$ where we assume that $I = N_n$, $J = N_m$ and $K = N_s$. The membership matrices of P, Q and R are denoted by $P = [p_{ij}]$, $Q[Q_{jk}]$, $R = [r_{ik}]$ where $p_{ij} \in P(x_i, y_j)$, $q_{jk} \in Q(y_j, z_k)$, $r_{ik} \in R(x_i, z_k)$ $\forall$ $i \in I(= N_n)$, $j \in J(= N_m)$ and $k \in K(= N_s)$. This clearly implies real number belongs to the P Q and R from the unit interval $[0,1]$. Assume now that three relations constraints each in such a way that

$$P_0 Q = R \tag{1}$$

where $_0$ denotes max-min composition. This means that

$$\max_{j \in J} \min (p_{ij}, q_{jk}) = r_{ik}$$

$$\forall i \in I \text{ and } k \in K \tag{2}$$

That is the matrix (1) encompasses $n \times s$ simultaneous equations of the form (2) when two of the components in each equations are given and one is unknown these equations are referred to a fuzzy relation equations. When matrices P and Q are given, the matrix R is to determined from (1). The problem is trival, it is solved simply by performing the max-min multiplication like operations on P and Q as defined by (2). This solution in this case exists and is unique. The problem becomes far from trival when one of the two matrices on the left side of (1) is unknown. In this case the solutions are guaranteed neither to exists nor to be unique. Since R in (1) obtained by composing P and Q; it is suggestive to view the problem of determining P (or alternatively Q) from R to Q (or alternatively R and P) as a decomposition of R with respect to Q (or P) assume that we have a method for solving (1) only for the first decomposition problem (given Q and R). Then we can directly utilize this method for solving the second decomposition problem as well. We simply write (1) in the form

$$Q^{-1}{}_{0}\ P^{-1} = R^{-1} \tag{3}$$

employing transported matrices. We can solve (3) for Q^{-1} by the method and then obtain the solution of (1) by $(Q^{-1})^{-1} = Q$.

2.2 Fuzzy Inverse Equations

A fuzzy relation $R^{-1} \subseteq Y \times X$ is called the inverse of the fuzzy relation $R \subseteq X \times Y$ which is defined as $R^{-1}(y,x) = R(x,y) \forall x \in X\ and\ \forall y \in Y$ for all pairs $(y,x) \in Y \times X$ and $\mu_{R^{-1}}(y,x) = \mu_R(x,y)$ (4)

where $\mu_{R^{-1}}$ is the membership function of R^{-1} . R^{-1} is defined as $R^{-1} = R^t\ and\ (R^{-1})^{-1} = R$.

3. Lattice

A lattice is a partially ordered set (poset) L in which any two elements x and y have a greatest lower bound (inf) denoted by $x \vee y = \max(x,y)$ and least upper bound (sup) denoted by .

3.1 A Brouwerian Lattice

A brouwerian lattice is a lattice L [5] in which for any given elements a and b , the set of all $x \in L$ such that $a \wedge x \leq b$ contains greatest element, denoted $a\ \alpha\ b$, called relative pseudo complement of a in b .

3.2 α Operator

For any given a in b in lattice $L \in [0,1], \alpha$ operator as

$$a\ \alpha\ b = \begin{cases} 1\ if\ a \leq b \\ b\ if\ a > b \end{cases} \tag{5}$$

It is also known as Sanchez operator.

Properties:

1. If $b = 0$ then $a\ \alpha\ b$ will be given as:

$$a\ \alpha\ 0 = \begin{cases} 1\ if\ a = b \\ 0\ if\ a > b \end{cases} \tag{6}$$

2. If $a = 0$ then $a\ \alpha\ b$ will be given as:
$0\ \alpha\ b = 1$
3. If $b = 1$ then $a\ \alpha\ b$ will be given as:
$a\ \alpha\ 1 = 1$
4. If $a = 1$ then $a\ \alpha\ b$ will be given as:
$1\ \alpha\ b = b$
5. α operator is not commutative.
$a\ \alpha\ b \neq b\ \alpha\ a$
6. α operator is not associative.
$a\ \alpha\ (b\ \alpha\ c) \neq (a\ \alpha\ b)\alpha\ c$

3.3 Composition of the @ Operator type

Consider two fuzzy relations $R \subseteq X \times Y$ and $Q \subseteq Y \times Z$. Relationship between these two fuzzy relations when using @ composition is defined as $R@\,Q \subseteq X \times Z$ with the membership function as

$$\mu_{R@Q}(x,z) = \wedge_{y\in Y}\{\mu_R(x,y)@\mu_Q(y,z)\}$$

$$\forall x\ \in X, y \in Y\ and\ z \in Z \qquad (7)$$

Properties:

$a\wedge(a\ @\ b) = a\wedge b \leq b$ (8)
$a\ \alpha\ b \geq b$ (9)
$a\ \alpha\ (b \vee c) \geq a\ @\ b$ (10)
$a\ @\ (a \wedge b\ ) \geq b$ (11)
$a\ \alpha\ (b \vee c) \geq a\ @\ c$ (12)

3.4 Determination of Maximal Q

For determination of maximal matrix Q , there is some lemma as given below:

Lemma 1:

If we have two fuzzy relations $R \subseteq X \times Y$ and $Q \subseteq Y \times Z$, then the following result will hold:

$Q \subseteq R^{-1}@\ (R\ _0\ Q)$ (13)
where $_0$ denotes the max-min composition and @ is the composition by α operator.

Proof:

Let $A = R^{-1} @ (R \,_0\, Q) \subseteq Y \times Z$. Then by using (4) and (12) we have

$\mu_A(y,z) = \wedge_{x \in X}\{\mu_{R^{-1}}(y,x) \alpha\, \mu_{R_0 Q}(x,z)\}$

$= \wedge_{x \in X}\{\mu_R(x,y) \alpha\, \mu_{R_0 Q}(x,z)\}$

$= \wedge_{x \in X}(\mu_R(x,y) \alpha\, (\vee_{t \in Y}\{\mu_R(x,t) \wedge \mu_Q(t,z)\}))$

$\wedge_{x \in X}(\mu_R(x,y) \alpha (\mu_R(x,y) \wedge \mu_Q(y,z) \vee_{t \in Y \neq y}$
$\left(\mu_R(x,t) \wedge \mu_Q(t,z)\right))$

So it becomes

$\mu_A(y,z) \geq \wedge_{x \in X}\{\mu_R(x,y) \alpha \left(\mu_R(x,y) \wedge \mu_Q(y,z)\right)\}$

as we know that

$$a \,\alpha\, (a \wedge b) \geq b \quad (14)$$

then by using (14) we have

$$\mu_A(y,z) \geq \mu_Q(y,z) \forall y \in Y \text{ and } z \in Z$$

Lemma 2:

Assume that we have two fuzzy relations $R \subseteq X \times Y$ and $T \subseteq Y \times Z$ then the following inclusion holds: `

$$R \,_0\, (R^{-1} @ T) \subset T \quad (15)$$

where $_0$ denotes the max-min composition and @ is the composition of α operator.

Lemma 3:

Consider two fuzzy relations $R \subseteq X \times Y$ and $Q \subseteq Y \times Z$ then the following inclusion holds:

$$R \subseteq (Q @ (R \,_0\, Q)^{-1})^{-1} \quad (16)$$

Lemma 4:

Consider two fuzzy relations $Q \subseteq Y \times Z$ and $T \subseteq X \times Z$ then the following inclusion holds:

$$(Q @ T^{-1})^{-1} \,_0\, Q \subset T \quad (17)$$

Theorem 1:

Let $R \subseteq \mathrm{X} \times \mathrm{Y}$ and $T \subseteq \mathrm{X} \times \mathrm{Z}$ be two fuzzy relations, $S(Q)$ be the set of fuzzy relations $Q \in \mathrm{Y} \times \mathrm{Z}$ such that $R\ _0\ Q = T$. Here ∇ denotes the maximal solution. $S(Q) = \{Q \in Y \times \mathrm{Z} | R\ _0\ Q = \mathrm{T}\} \neq \emptyset$, if and only if $R^{-1}@T \in S(Q)$ then $R^{-1}@T$ is the greatest element in $S(Q)$.

Proof:

Let $S(Q)^* = \{Fuzzy\ Q \in (Y \times Z) R\ _0\ Q \subseteq \mathrm{T}\}$ and $S(R)^* \neq \emptyset$ because of the null relation $0(y,z) = 0\ \forall (y,z) \in Y \times Z \in S(Q)^*$

let $Q \subseteq \mathrm{S(R)}^*: R\ _0\ Q = \mathrm{T}$

then we have $R^{-1}@(R\ _0\ Q) \subseteq \mathrm{R}^{-1}@\mathrm{T}$ but from (13) we have $Q \subset \mathrm{R}^{-1}@(R\ _0\ Q)$ then $Q \subset \mathrm{R}^{-1}@T$ now $\mathrm{R}^{-1}@T \in S(Q)$. Then it shows that $\mathrm{R}^{-1}@T \in S(Q)^*$, then $\mathrm{R}^{-1}@T$ will be the greatest element in $S(Q)^*$. Hence $\mathrm{R}^{-1}@T$ be the greatest element in $S(Q)^*$. Then

$Q^{\nabla} = R^{-1}@\text{Error! Bookmark not defined.}T$

(18) which is the maximum relation Q satisfying the equation $R\ _0\ Q = T$.

3.5 Necessary condition for the existence of Q^{∇}

The necessary condition for the existence of Q^{∇} satisfying the FRE (1) is

$$\mu_T(x,z) \leq {}_{y \in Y}\vee \mu_R(x,y) \forall x \in X and\ z \in Z \quad (19)$$

Theorem 2:

Let $Q \subseteq \mathrm{Y} \times \mathrm{Z}$ and $T \subseteq \mathrm{X} \times \mathrm{Z}$ be the two fuzzy relations, $S(R)$ be the set of fuzzy relations $R \in \mathrm{X} \times \mathrm{Y}$ such that $R\ _0\ Q = T$.

$S(R) = \{Fuzzy\ R \in X \times Y | R\ _0\ Q = T\} \neq \emptyset$, if and only if $(Q@T^{-1})^{-1} \in S(R)$ and it is the greatest element in $S(R)$.

Proof:

Let $S(R)^* = \{Fuzzy\ R \in (X \times Y) | R\ _0\ Q \subseteq \mathrm{T}$ and $S(R)^* \neq \emptyset$ because of the null relation $0(x,y) = 0\ \forall (x,y) \in Y \times Z \in S(R)^*$.

Let $R \subseteq \mathrm{S(R)}^*: R\ _0\ Q = \mathrm{T}$, then we have $(Q@(R\ _0\ Q)^{-1})^{-1} \subseteq (Q@T^{-1})^{-1}$,

but from Lemma 3, we have $R \subset (\mathrm{Q}@(R\ _0\ Q)^{-1})^{-1}$.

Then it shows that $R \subset (Q@T^{-1})^{-1}$,

now from theorem we have $(Q@T^{-1})^{-1} \in S(R)$.

Then it shows that $(Q@T^{-1})^{-1} \in S(R)^*$, Hence $(Q@T^{-1})^{-1}$ be the greatest element in $S(R)^*$.

so $R^{\nabla} = (Q@T^{-1})^{-1}$ (20)

which is the maximum relation for R satisfying the equation $R_0 Q = T$.

3.5 Necessary condition for existence of R^{∇}

The necessary condition for the existence of R^{∇} satisfying the FRE is

$\mu_T(x,z) \leq {}_{y\in Y}\gamma\mu_Q(y,z) \forall x \in X and\ z \in Z$ (21)

4. Analysis of the Performances of Faculties Based on Student Feedback Using Fuzzy Operators

In recent years feedback system is very essential for many institutes. This system can help us to improve the teacher's performance, techniques, teaching style and many more. Feedback system contains various factor like "Timeliness", "Communication skill", "Control of the class" and many more. These factors are in linguistic form. In this paper we denote the factors as "Performance" and "Significance". Significance of performance usually has a strong effect on a project e.g. if a factor has very high performance but the significance of this factor is nil then one can solve this problem by finding the significance of the performance using FRE. Performance is described by primary membership values belonging to a base set $Y = [0,1]$. Performance values are given by a group of experts.

Table 1: Performance factors are measured by their primary membership values

	y_1 0.0	y_2 0.1	y_3 0.2	y_4 0.3	y_5 0.4	y_6 0.5	y_7 0.6	y_8 0.7	y_9 0.8	y_{10} 0.9	y_{11} 1.0
Superior	0	0	0	0	0	0.13	0.25	0.35	0.8	0.9	1.0
Average	0	0.03	0.12	0.6	0.8	1.0	0.8	0.6	0.12	0.03	0
Poor	1.0	0.7	0.46	0.3	0.25	0.04	0	0	0	0	0

Significance is described by primary membership values to a base set $z = [0,1]$. The significance values are collected by some group of experts.

Table 2: Significance factors are measured by their primary membership values

	z_1 0.0	z_2 0.1	z_3 0.2	z_4 0.3	z_5 0.4	z_6 0.5	z_7 0.6	z_8 0.7	z_9 0.8	z_{10} 0.9	z_{11} 1.0
Critical	0	0	0	0	0	0	0.04	0.3	0.45	0.8	1.0
Important	0	0.10	0.25	0.5	0.7	1.0	0.7	0.5	0.25	0.10	0

Now we define secondary linguistic values for performance factors which are indeed-superior, rather superior, above superior, below average, very-poor and for secondary linguistic values of significance factor may be indeed critical, rather-critical, very-important, rather-important, not-important. In order to define these secondary linguistic values, first we denote the primary linguistic values as B(x). So the secondary values are defined by using the Baldwin approach as shown below:

$$indeed - B(x) = int(B(x))$$

$$rather - B(x) = \sqrt{B(x)}$$

$$very - B(x) = (B(x))^2$$

$$above - B(x) = \begin{cases} \neg B(x), if y \geq 0.5 \\ 0, if\ y < 0.5 \end{cases}$$

$$below - B(x) = \begin{cases} \neg B(x), if y \leq 0.5 \\ 0, if\ y > 0.5 \end{cases}$$

The function increases the high membership values and decreases the low membership values. So the secondary values for performance and significance are given below:

Table 3: Performance factors are measured by their secondary membership values

	y_1 0.0	y_2 0.1	y_3 0.2	y_4 0.3	y_5 0.4	y_6 0.5	y_7 0.6	y_8 0.7	y_9 0.8	y_{10} 0.9	y_{11} 1.0
Indeed-superior	0	0	0	0	0	0.03	0.15	0.25	0.92	0.98	1.0
Rather-superior	0	0	0	0	0	0.36	0.5	0.59	0.89	0.95	0
Above-average	0	0	0	0.4	0.2	0	0.2	0.4	0	0	0
Below-average	1	0.97	0.88	0	0	0	0	0	0.88	0.97	1
Very-poor	1	0.49	0.21	0.09	0.06	0.0016	0	0	0	0	0

Table 4: Significance factors are measured by their secondary membership values

	y_1 0.0	y_2 0.1	y_3 0.2	y_4 0.3	y_5 0.4	y_6 0.5	y_7 0.6	y_8 0.7	y_9 0.8	y_{10} 0.9	y_{11} 1.0
Indeed-critical	0	0	0	0	0	0	0.0032	0.18	0.41	0.92	1
Rather-critical	0	0	0	0	0	0	0.2	0.55	0.67	0.89	1
Very-important	0	0.01	0.063	0.25	0.49	1	0.49	0.25	0.063	0.01	0
Rather-important	0	0.32	0.5	0.71	0.84	1	0.84	0.71	0.5	0.32	0
Not-important	1	0.9	0.75	0.5	0.3	0	0.3	0.5	0.75	0.9	1

We have taken average results of the performance of feedback of teachers which are given by the students. The results of performance factors which are given by the students are in linguistic form. Factors relating with feedback system are as follows:

C_1 → Timeliness
C_2 → Attempt to complete syllabus & adherence to lecture plan
C_3 → Whether well performed & enough knowledgeable about the topic
C_4 → Communication skill
C_5 → Control of the class
C_6 → Involvement in doubt clearance
C_7 → Present the material clearly in the class
C_8 →Regular checking of class assignments
C_9 → proper experimental guidance
C_{10} → Responsibility
C_{11} → Teacher is approachable outside the class
C_{12} → Assessment as a guide and well wisher

Table 5: Performance of the faculties in linguistic form

Factor	Faculty 1	Faculty 2	Performance of Faculty 3	Faculty 4	Faculty 5	Significance
C_1	Superior	Average	Indeed-superior	Average	Below-average	Very-important
C_2	Superior	Below-average	Superior	Average	Average	Indeed-critical
C_3	Superior	Average	Indeed-superior	Poor	Below-average	Very-important
C_4	Indeed-superior	Average	Average	Poor	Poor	Very-important
C_5	Superior	Below-average	Above-average	Average	Above-average	Important
C_6	Above-average	Average	Average	Average	Poor	Rather-important
C_7	Average	Above-average	Superior	Above-average	Poor	Important
C_8	Superior	Below-average	Indeed-superior	Below-average	Very-poor	Indeed-critical
C_9	Above-average	Average	Superior	Above-average	Poor	Rather-critical
C_{10}	Superior	Average	Superior	Average	Average	Important
C_{11}	Superior	Below Average	Indeed-superior	Poor	Above-average	Indeed-critical
C_{12}	Indeed-superior	average	Superior	average	Above-average	Rather-critical

Now we have calculated maximum fuzzy relation (Q_{ij}^{∇}) among performance (R_{ij}) of projects $i = 1,2,3,4,5$ relative to factors $j = 1,2,\dots,12$ and the significance (T_{ij}) of factor j, satisfying the FRE $T_{ij} = R_{ij} \circ Q_{ij}$, where $R_{ij}^{-1} @ T_j$ is the maximum fuzzy relation.

Now for the 1st faculty $i = 1$

$R_{11} = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0.13\ \ 0.25\ \ 0.35\ \ 0.8\ \ 0.9\ \ 1.0]$;

$T_1 = [0\ \ 0.01\ \ 0.063\ \ 0.25\ \ 0.49\ \ 1\ \ 0.49\ \ 0.25\ \ 0.63\ \ 0.01\ \ 0]$

$$R_{11}^{-1}@T_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0.01 & 0.063 & 1 & 1 & 1 & 1 & 1 & 0.063 & 0.01 & 0 \\ 0 & 0.01 & 0.063 & 1 & 1 & 1 & 1 & 1 & 0.063 & 0.01 & 0 \\ 0 & 0.01 & 0.063 & 0.25 & 1 & 1 & 1 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0.01 & 0.063 & 0.25 & 0.49 & 1 & 0.49 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0.01 & 0.063 & 0.25 & 0.49 & 1 & 0.49 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0.01 & 0.063 & 0.25 & 0.49 & 1 & 0.49 & 0.25 & 0.063 & 0.01 & 0 \end{bmatrix}$$

Similarly calculation for $j = 1,2,\ldots,12$ and $\cap_{j=1,2,\ldots,12} Q_{ij} = \wedge_{j=1,2,\ldots,12} Q_{ij}$, where $\wedge$ denotes the 'min' function.

$$Q_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0.01 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0.01 & 0 \\ 0 & 0.01 & 0.063 & 1 & 1 & 1 & 1 & 1 & 0.063 & 0.01 & 0 \\ 0 & 0.01 & 0.063 & 0.25 & 0.49 & 1 & 0.49 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0.01 & 0.063 & 0.25 & 0.49 & 1 & 0.49 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \end{bmatrix}$$

Similarly we have calculated for the other faculties for $i = 2,3,4,5$ and determine the value for Q_2, Q_3, Q_4, Q_5

$$Q_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.25 & 0.10 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.25 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.25 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.25 & 0.10 & 0 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0.01 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0.01 & 0 \\ 0 & 0.01 & 0.063 & 1 & 1 & 1 & 1 & 1 & 0.063 & 0.01 & 0 \\ 0 & 0.01 & 0.063 & 0.25 & 0.49 & 1 & 0.49 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0.01 & 0.063 & 0.25 & 0.49 & 1 & 0.49 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \end{bmatrix}$$

$$Q_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.41 & 0.063 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.41 & 0.063 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.41 & 0.063 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.41 & 0.063 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.41 & 0.063 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.41 & 0.063 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.41 & 0.063 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.41 & 0.063 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.41 & 0.063 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.41 & 0.92 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.41 & 0.92 & 1 \end{bmatrix}$$

$$Q_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.25 & 0.10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.18 & 0.25 & 0.10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0032 & 0.25 & 0.063 & 0.01 & 0 \\ 0 & 0.01 & 0.03 & 0.25 & 0.49 & 1 & 0.49 & 0.25 & 0.063 & 0.01 & 0 \end{bmatrix}$$

If the significance of each factor is defined by the administration as 'very-important' then we have solved the relational equation for $i = 1,2,\ldots,8$. Now we compute the $y^{-1} = Q_i @ (very - important)^{-1}$ for $i = 1,2,\ldots,8$ as given bellow.

For faculty 1

$$y_1^{-1} = Q_i @(very - important)^{-1}$$
$$= [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0.25\ \ 0.18\ \ 0.18\ \ 0.18\ \ \ 0.18\ \ \ 0.18]^T$$

$y_1 =$
$[1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0.25\ \ 0.18\ \ 0.18\ \ 0.18\ \ \ 0.18\ \ \ 0.18]$

For faculty 2

$$y_2 = \begin{bmatrix} 0.25\ \ 0.25\ \ 0.18\ \ 0.25\ \ 0.25\ \ 0.25\ \ 0.25\ \ 0.25\ \ 0.25 \\ 0.18\ \ 0.25\ \ 0.25 \end{bmatrix}$$

For faculty 3

$$y_3 = [1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 0.25\ \ 0.18\ \ 0.18\ \ 0.18\ \ 0.18\ \ 0.18]$$

For faculty 4

$$y_4 = \begin{bmatrix} 0.18\ \ 0.18\ \ 0.18\ \ 0.18\ \ 0.18\ \ 0.18\ \ 0.18\ \ 0.18 \\ 0.18\ \ 0.18\ \ 0.18 \end{bmatrix}$$

For faculty 5

$$y_5 = \begin{bmatrix} 0.18\ \ 0.18\ \ 0.18\ \ 0.18\ \ 0.18\ \ 0.18\ \ 0.18\ \ 0.18 \\ 0.25\ \ 0.25\ \ 0.25 \end{bmatrix}$$

4.1 De-fuzzification Using Mean of Maximum Method

De-fuzzification methods are usually used to rank the projects. Mean of maximum method is one of the methods for de-fuzzifying

$$d_{MM}(F) = \sum_{y_k \in M} (y_k)/|M| \quad (22)$$

where $M = \{y_k \mid F(y_k) = hgt(F)\}$

$F(y)$ is the membership function of a fuzzy set F. $hgt(F)$ is the maximum membership value of the fuzzy set F and $|M|$ is the cardinality of M .

Now for faculty 1, $M_1 = \{0, 0.1,02,0.3,0.4\}$

for faculty 2, $M_2 = \{0,0.1,0.3,0.4,0.5,0.6,0.7,0.9,1\}$

for faculty 3, $M_3 = \{0,0.1,0.2,0.3,0.4\}$

for faculty 4, $M_4 = \{0,0.1,0.2,0.3,0.4,0.5,0.65,0.7,0.8,0.9,1\}$

for faculty 5, $M_5 = \{0,.8,0.9,1\}$

Mean of maximum method as given below

$M_{M_1} = 0.65$
$M_{M_2} = 0.5$
$M_{M_3} = 0.65$
$M_{M_4} = 0.45$
$M_{M_5} = 0.46$
M_{M_1} and M_{M_3} gives us the best result, so faculty 1 and faculty 3 shows their best performance during session.

5. Conclusion

In this paper we have distinguished an optimal solution to the problem of analysis of the performances of student feedback on faculty teaching using various fuzzy operators on fuzzy relation equations. The result obtained has helped us to pick the best performance of a faculty among the various factors which is in terms of linguistic form and FREs have applied on this factors to find the best outcome which is ideal in this problem.

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