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Some Ranking Indexes of Stochastic Orders

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Abstract

In this paper we recall some of the known stochastic orders and the shifted version of them and we discuss their relations and properties. Further, we obtain some applications of proportional likelihood ratio ordering, fuzzy hazard rate ordering and mean inactivity ordering.

Keywords: *Fuzzy random variables, Fuzzy likelihood ratio order, Fuzzy hazard rate order, Mean inactivity time order, Shifted stochastic orders.*

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1. Introduction

Stochastic orders have been proven to be very useful in applied probability, statistics, reliability, operation research, economics and other fields. Various types of stochastic orders and associate properties have been developed rapidly over the years [1-12].

A lot of research works have done on hazard rate and reversed hazard rate orders due to their properties and applications in the various sciences, for example hazard rate order is a useful tool in reliability theory and reversed hazard rate order is defined via stochastic comparison of inactivity time. Ramos-Romero and Sordo-Diaz [9] introduced a new stochastic order between two absolutely continuous random variables and called it Proportional Hazard Rate order (PHR), which is closely related to the usual Hazard Rate order. The PHR order can be used to characterize random variables whose logarithms have log-concave (log-convex) densities. Many income random variables satisfy this property and they are said to have the increasing proportional Hazard Rate order (*IPHR*) and decreasing proportional Hazard Rate Order (*DPHR*) properties. As an application Romero and Sordo-Diaz [9] showed that the *IPHR* and *DPHR* properties are sufficient conditions for the Lorenz ordering of truncated distributions.

Shifted stochastic orders, which are useful tools for establishing interesting inequalities, have been also introduced and studied, like the up likelihood ratio order, the down likelihood ratio order, the up hazard rate order and the down hazard rate order.

In this paper we recall the proportional state of stochastic orders and the shifted version of them and we obtain some applications of proportional Hazard Rate order.

2. Preliminaries

2.1 Fuzzy Numbers

Let X be the universal set, then a fuzzy subset (briefly, a fuzzy set) $\bar{x}$ of X is defined by its membership function $\mu_{\bar{x}}$: X → [0, 1]. We denote the α –cuts of $\tilde{x}$ by $\tilde{x}_\alpha = \{x : \mu_{\tilde{x}}(x) \geq \alpha\}$ for all *a* in [0, 1], while its closure is the set $\{x : \mu_{\tilde{x}}(x) > 0$ }.

A fuzzy set is called a normal fuzzy set if there exists x in X such that $\mu_{\bar{x}}$ (x) = 1 and it is called a convex fuzzy set if $\mu_{\tilde{x}}$ (λ$x + (1-\lambda)y$) ≥ min$\{\mu_{\tilde{x}}(x), \mu_{\tilde{x}}(y)\}$ for every x, y in X and λ $\in [0,1]$. A fuzzy set $\tilde{x}$ of the set ***R*** of real numbers is called a fuzzy number if it is a normal and convex fuzzy set and its α –cuts are bounded for all α $\in [0,1]$. In addition, if $\tilde{x}$ is a fuzzy number and the support of its membership function $\mu_{\tilde{x}}$ is compact, then $\tilde{x}$ is called a bounded fuzzy number.

If $\tilde{x}$ is a closed and bounded fuzzy number with $\tilde{x}_\alpha^L = min\{x : x \in \tilde{x}_\alpha\}$ and its membership function is strictly increasing on the interval $[x_\alpha^L, x_1^L]$ and strictly decreasing on the interval $[x_\alpha^U, x_1^U]$, then $\tilde{x}$ is called a canonical fuzzy number.

2.2 Fuzzy Random Variables

Let f be a non negative real function with domain the universal set X and let S_X be the support of f. Then a fuzzy number $\tilde{x}$ with membership function $\mu_{\tilde{x}}(r)$ can be induced by any real number $x \in S_X$ such that $\mu_{\tilde{x}}(x) = 1$ and $\mu_{\tilde{x}}(r) < 1$ for $r \neq x$ .We denote the set of all fuzzy numbers induced by the real number $x \in S_X$ by F(S_X).

The relation $\tilde{}$ on F(S_X) is defined as $\tilde{x}_1 \sim \tilde{x}_2$ if and only if $\tilde{x}_1$ and $\tilde{x}_2$ are induced by the same real number x . Then $\tilde{}$ is an equivalence relation, which induces the equivalence classes $[\tilde{x}] = \{\tilde{a} : \tilde{a} \sim \tilde{x}\}$. The set (F($S_X$)/$\tilde{}$) is called a fuzzy real number system. In practice, we take only one element $\tilde{x}$ from each equivalence class $[\tilde{x}]$ to form the fuzzy real number system (F(S_X)/$\tilde{}$). If the fuzzy real number system (F(S_X)/$\tilde{}$) consists of all the canonical fuzzy real numbers, then we call (F(S_X)/$\tilde{}$) the canonical fuzzy real number system.

Let X be a random variable with support S_X and $F(S_X)$ be the set of all canonical fuzzy numbers induced by the real numbers in S_X. Then a fuzzy random variable is a function X: Y $\rightarrow$ $F(S_X)$, where for all $\alpha \in [0,1]$.

$$\{(\omega, x) : \omega \in \psi, x \in \tilde{X}_\alpha(\omega)\} \in F \times \mathcal{B}$$

Since F(S_X) is the support of the fuzzy random variable $\tilde{X}$, each α –cut set of depends on the random variable X .

2.3 Definition: Let F(R) be a canonical fuzzy real number system. Then $\tilde{X}$ is a fuzzy random variable if and only if $\tilde{X}_\alpha^L$ and are ordinary random variables for all $\alpha \in$ [0,1].

3. Fuzzy Likelihood Ratio Order

Let X be a non negative fuzzy random variable with density function $f(\tilde{x})$ and cumulative distribution function $\overline{F}(\tilde{x})$ respectively, and $\tilde{x}$ be a fuzzy random variables induced by X. The fuzzy function $\tilde{r}_{\tilde{F}}(\tilde{x})$ is a likelihood ratio order of fuzzy random variables $\tilde{x}$, whenever its membership function is given by

$$\mu_{\tilde{f}_{\tilde{x}}}(y) = \left(\sup_{0 \le \alpha \le 1}\right) \alpha I \{ \tilde{r}(\tilde{x})_{\alpha}^{L}, \}(y)$$

Where, $f(\tilde{x})_{\alpha}^{L}$ =

$$\min \left\{ \left\{ \left(\min_{\alpha \le \beta \le 1} \right) f(\tilde{x}) : x = \tilde{x}_{\beta}^{L} \right\}, \left\{ \left(\min_{\alpha \le \beta \le 1} \right) f(\tilde{x}) : x = \tilde{x}_{\beta}^{U} \right\} \right\},$$

$f(\tilde{x})_{\alpha}^{U}$ =

$$\max \left\{ \left\{ \left(\max_{\alpha \le \beta \le 1} \right) f(\tilde{x}) : x = \tilde{x}_{\beta}^{L} \right\}, \left\{ \left(\max_{\alpha \le \beta \le 1} \right) f(\tilde{x}) : x = \tilde{x}_{\beta}^{U} \right\} \right\},$$

Such that the interval, $f(\tilde{x})_{\alpha}^{L}$ and $f(\tilde{x})_{\alpha}^{U}$ will contain all of the cumulative distribution function for $\beta \ge \alpha$. We note that $x_{\alpha}^{L}, x_{1}^{L}, x_{\alpha}^{U}, x_{1}^{U}$

Let X and Y be continuous non negative fuzzy random variables with density functions f and g respectively, we propose four relations to compare fuzzy likelihood ratio order of X and Y as follows:

(1) X $\le_{FLR1}$ Y if,

$$\min \left\{ \left(\min_{\alpha \le \beta \le 1} \right) f(\tilde{x})_{\alpha}^{L}, \left(\min_{\alpha \le \beta \le 1} \right) g(\tilde{x})_{\alpha}^{L} \right\} \le \min \left\{ \left(\min_{\alpha \le \beta \le 1} \right) f(\tilde{y})_{\alpha}^{U}, \left(\min_{\alpha \le \beta \le 1} \right) g(\tilde{y})_{\alpha}^{U} \right\}$$

And

$$\max \left\{ \left(\max_{\alpha \le \beta \le 1} \right) f(\tilde{x})_{\alpha}^{L}, \left(\max_{\alpha \le \beta \le 1} \right) g(\tilde{x})_{\alpha}^{L} \right\} \le \max \left\{ \left(\max_{\alpha \le \beta \le 1} \right) f(\tilde{y})_{\alpha}^{U}, \left(\max_{\alpha \le \beta \le 1} \right) g(\tilde{y})_{\alpha}^{U} \right\}$$

(2) X $\le_{FLR2}$ Y if,

$$\min \left\{ \left(\min_{\alpha \le \beta \le 1} \right) f(\tilde{x})_{\alpha}^{L}, \left(\min_{\alpha \le \beta \le 1} \right) g(\tilde{x})_{\alpha}^{U} \right\} \le \min \left\{ \left(\min_{\alpha \le \beta \le 1} \right) f(\tilde{y})_{\alpha}^{L}, \left(\min_{\alpha \le \beta \le 1} \right) g(\tilde{y})_{\alpha}^{U} \right\}$$

And

$$\max \left\{ \left(\max_{\alpha \le \beta \le 1} \right) f(\tilde{x})_{\alpha}^{L}, \left(\max_{\alpha \le \beta \le 1} \right) g(\tilde{x})_{\alpha}^{U} \right\} \le \max \left\{ \left(\max_{\alpha \le \beta \le 1} \right) f(\tilde{y})_{\alpha}^{L}, \left(\max_{\alpha \le \beta \le 1} \right) g(\tilde{y})_{\alpha}^{U} \right\}$$

(3) X $\le_{FLR3}$ Y if,

$$\min \{(_\alpha \le \beta \le 1^{min}) f(\tilde{x})_\alpha^U, (_\alpha \le \beta \le 1^{min} g(\tilde{x})_\alpha^U \le \min \{(_\alpha \le \beta \le 1^{min}) f(\tilde{y})_\alpha^U, (_\alpha \le \beta \le 1^{min} g(\tilde{y})_\alpha^U$$

And

$$\max \{(_\alpha \le \beta \le 1^{max}) f(\tilde{x})_\alpha^U, (_\alpha \le \beta \le 1^{max} g(\tilde{x})_\alpha^U \le \max \{(_\alpha \le \beta \le 1^{max}) f(\tilde{y})_\alpha^U, (_\alpha \le \beta \le 1^{max} g(\tilde{y})_\alpha^U$$

(4) X $\le_{FLR4}$ Y if,

$$\min \{(_\alpha \le \beta \le 1^{min}) f(\tilde{x})_\alpha^L, (_\alpha \le \beta \le 1^{min} g(\tilde{x})_\alpha^L \le \min \{(_\alpha \le \beta \le 1^{min}) f(\tilde{y})_\alpha^L, (_\alpha \le \beta \le 1^{min} g(\tilde{y})_\alpha^L$$

And

$$\max \{(_\alpha \le \beta \le 1^{max}) f(\tilde{x})_\alpha^L, (_\alpha \le \beta \le 1^{max} g(\tilde{x})_\alpha^L \le \max \{(_\alpha \le \beta \le 1^{max}) f(\tilde{y})_\alpha^L, (_\alpha \le \beta \le 1^{max} g(\tilde{y})_\alpha^L$$

For all X ≤ Y, $\alpha \in [0,1]$.

3.2 Up Proportional Fuzzy Likelihood Ratio Order

Suppose that X and Y are two continuous non negative fuzzy random variables with density functions f and g respectively, we propose four relations to compare Up proportional fuzzy likelihood ratio order of X and Y as follows.

(1) X $\le_{PFLR1}$ Y if,

$$\min \{(_\alpha \le \beta \le 1^{min}) [\![f[\lambda\tilde{x}]\!]_\alpha^L - \frac{t}{\lambda\tilde{x}_\alpha^L} \ge t \Big], (_\alpha \le \beta \le 1^{min} g [\![[\lambda\tilde{x}]\!]_\alpha^L - \frac{t}{\lambda\tilde{x}_\alpha^L} \ge t \Big] \ge \min \{(_\alpha \le \beta \le 1^{min}) f(\tilde{y})_\alpha^U, (_\alpha \le \beta \le 1^{min} g(\tilde{y})_\alpha^U$$

And

$$\max \{(_\alpha \le \beta \le 1^{max}) [\![f[\lambda\tilde{x}]\!]_\alpha^L - \frac{t}{\lambda\tilde{x}_\alpha^L} \ge t \Big], (_\alpha \le \beta \le 1^{min} g [\![[\lambda\tilde{x}]\!]_\alpha^L - \frac{t}{\lambda\tilde{x}_\alpha^L} \ge t \Big] \ge \max \{(_\alpha \le \beta \le 1^{max}) f(\tilde{y})_\alpha^U, (_\alpha \le \beta \le 1^{max} g(\tilde{y})_\alpha^U$$

(2) X $\le_{PFLR2}$ Y if,

$$\min \{(_\alpha \le \beta \le 1^{min}) [\![f[\lambda\tilde{x}]\!]_\alpha^L - \frac{t}{\lambda\tilde{x}_\alpha^L} \ge t \Big], (_\alpha \le \beta \le 1^{min} g [\![[\lambda\tilde{x}]\!]_\alpha^U - \frac{t}{\lambda\tilde{x}_\alpha^U} \ge t \Big] \ge \min \{(_\alpha \le \beta \le 1^{min}) f(\tilde{y})_\alpha^L, (_\alpha \le \beta \le 1^{min} g(\tilde{y})_\alpha^U$$

And

$$\max\{(_\alpha \leq \beta \leq 1^{max})\,[\![f[\lambda\tilde{x}]\!]_\alpha^L - \frac{t}{\lambda\tilde{x}_\alpha^L} \geq t\Big],\ (_\alpha \leq \beta \leq 1^{min}\, g\,[\![[\lambda\tilde{x}]\!]_\alpha^U - \frac{t}{\lambda\tilde{x}_\alpha^U} \geq t\Big] \geq$$
$$\max\{(_\alpha \leq \beta \leq 1^{max}) f(\tilde{y})_\alpha^L,\ (_\alpha \leq \beta \leq 1^{max}\, g(\tilde{y})_\alpha^U$$

(3) X $\leq_{PFLR3}$ Y if,

$$\min\{(_\alpha \leq \beta \leq 1^{min})\,[\![f[\lambda\tilde{x}]\!]_\alpha^U - \frac{t}{\lambda\tilde{x}_\alpha^U} \geq t\Big],\ (_\alpha \leq \beta \leq 1^{min}\, g\,[\![[\lambda\tilde{x}]\!]_\alpha^U - \frac{t}{\lambda\tilde{x}_\alpha^U} \geq t\Big] \geq$$
$$\min\{(_\alpha \leq \beta \leq 1^{min}) f(\tilde{y})_\alpha^U,\ (_\alpha \leq \beta \leq 1^{min}\, g(\tilde{y})_\alpha^U$$

And

$$\max\{(_\alpha \leq \beta \leq 1^{max})\,[\![f[\lambda\tilde{x}]\!]_\alpha^U - \frac{t}{\lambda\tilde{x}_\alpha^U} \geq t\Big],\ (_\alpha \leq \beta \leq 1^{min}\, g\,[\![[\lambda\tilde{x}]\!]_\alpha^U - \frac{t}{\lambda\tilde{x}_\alpha^U} \geq t\Big] \geq$$
$$\max\{(_\alpha \leq \beta \leq 1^{max}) f(\tilde{y})_\alpha^U,\ (_\alpha \leq \beta \leq 1^{max}\, g(\tilde{y})_\alpha^U$$

(4) X $\leq_{PFLR3}$ Y if,

$$\min\{(_\alpha \leq \beta \leq 1^{min})\,[\![f[\lambda\tilde{x}]\!]_\alpha^L - \frac{t}{\lambda\tilde{x}_\alpha^L} \geq t\Big],\ (_\alpha \leq \beta \leq 1^{min}\, g\,[\![[\lambda\tilde{x}]\!]_\alpha^L - \frac{t}{\lambda\tilde{x}_\alpha^L} \geq t\Big] \geq$$
$$\min\{(_\alpha \leq \beta \leq 1^{min}) f(\tilde{y})_\alpha^L,\ (_\alpha \leq \beta \leq 1^{min}\, g(\tilde{y})_\alpha^L$$

And

$$\max\{(_\alpha \leq \beta \leq 1^{max})\,[\![f[\lambda\tilde{x}]\!]_\alpha^L - \frac{t}{\lambda\tilde{x}_\alpha^L} \geq t\Big],\ (_\alpha \leq \beta \leq 1^{min}\, g\,[\![[\lambda\tilde{x}]\!]_\alpha^L - \frac{t}{\lambda\tilde{x}_\alpha^L} \geq t\Big] \geq$$
$$\max\{(_\alpha \leq \beta \leq 1^{max}) f(\tilde{y})_\alpha^L,\ (_\alpha \leq \beta \leq 1^{max}\, g(\tilde{y})_\alpha^L$$

For all X ≤ Y, α ∈ [0,1].

3.3 Down Proportional Fuzzy Likelihood Ratio Order

Suppose that X and Y are two continuous non negative fuzzy random variables with density functions f and g respectively, we propose four relations to compare Down proportional fuzzy likelihood ratio order of X and Y as follows.

(1) X $\leq_{PFLR1}$ Y if,

$$\min\{(_\alpha \leq \beta \leq 1^{min}) f(\tilde{x})_\alpha^L,\ (_\alpha \leq \beta \leq 1^{min}\, g(\tilde{x})_\alpha^L \leq$$
$$\min(_\alpha \leq \beta \leq 1\{\ \square^{min}) f\left[\lambda\tilde{y}_\alpha^U - \frac{t}{\lambda\tilde{y}_\alpha^U} \geq t\right],\quad (_\alpha \leq \beta \leq 1^{min}\, g\left[\lambda\tilde{y}_\alpha^U - \frac{t}{\lambda\tilde{y}_\alpha^U} \geq t\right]$$

And

$$\max \{({}_{\alpha}\leq \beta \leq 1^{max}) f(\tilde{x})^{L}_{\alpha}, ({}_{\alpha}\leq \beta \leq 1^{max} g(\tilde{x})^{L}_{\alpha} \quad \leq$$

$$\min ({}_{\alpha}\leq \beta \leq 1\{\square^{max}) f\left[\lambda\tilde{y}^{U}_{\alpha} - \frac{t}{\lambda\tilde{y}^{U}_{\alpha}} \geq t\right], \quad ({}_{\alpha}\leq \beta \leq 1^{max} g\left[\lambda\tilde{y}^{U}_{\alpha} - \frac{t}{\lambda\tilde{y}^{U}_{\alpha}} \geq t\right]$$

(2) X $\leq_{PFLR2}$ Y if,

$$\min \{({}_{\alpha}\leq \beta \leq 1^{min}) f(\tilde{x})^{L}_{\alpha}, ({}_{\alpha}\leq \beta \leq 1^{min} g(\tilde{x})^{U}_{\alpha} \quad \leq$$

$$\min ({}_{\alpha}\leq \beta \leq 1\{\square^{min}) f\left[\lambda\tilde{y}^{L}_{\alpha} - \frac{t}{\lambda\tilde{y}^{L}_{\alpha}} \geq t\right], \quad ({}_{\alpha}\leq \beta \leq 1^{min} g\left[\lambda\tilde{y}^{U}_{\alpha} - \frac{t}{\lambda\tilde{y}^{U}_{\alpha}} \geq t\right]$$

And

$$\max \{({}_{\alpha}\leq \beta \leq 1^{max}) f(\tilde{x})^{L}_{\alpha}, ({}_{\alpha}\leq \beta \leq 1^{max} g(\tilde{x})^{U}_{\alpha} \quad \leq$$

$$\min ({}_{\alpha}\leq \beta \leq 1\{\square^{max}) f\left[\lambda\tilde{y}^{L}_{\alpha} - \frac{t}{\lambda\tilde{y}^{L}_{\alpha}} \geq t\right], \quad ({}_{\alpha}\leq \beta \leq 1^{max} g\left[\lambda\tilde{y}^{U}_{\alpha} - \frac{t}{\lambda\tilde{y}^{U}_{\alpha}} \geq t\right]$$

(3) X $\leq_{PFLR3}$ Y if,

$$\min \{({}_{\alpha}\leq \beta \leq 1^{min}) f(\tilde{x})^{U}_{\alpha}, ({}_{\alpha}\leq \beta \leq 1^{min} g(\tilde{x})^{U}_{\alpha} \quad \leq$$

$$\min ({}_{\alpha}\leq \beta \leq 1\{\square^{min}) f\left[\lambda\tilde{y}^{U}_{\alpha} - \frac{t}{\lambda\tilde{y}^{U}_{\alpha}} \geq t\right], \quad ({}_{\alpha}\leq \beta \leq 1^{min} g\left[\lambda\tilde{y}^{U}_{\alpha} - \frac{t}{\lambda\tilde{y}^{U}_{\alpha}} \geq t\right]$$

And

$$\max \{({}_{\alpha}\leq \beta \leq 1^{max}) f(\tilde{x})^{U}_{\alpha}, ({}_{\alpha}\leq \beta \leq 1^{max} g(\tilde{x})^{U}_{\alpha} \quad \leq$$

$$\min ({}_{\alpha}\leq \beta \leq 1\{\square^{max}) f\left[\lambda\tilde{y}^{U}_{\alpha} - \frac{t}{\lambda\tilde{y}^{U}_{\alpha}} \geq t\right], \quad ({}_{\alpha}\leq \beta \leq 1^{max} g\left[\lambda\tilde{y}^{U}_{\alpha} - \frac{t}{\lambda\tilde{y}^{U}_{\alpha}} \geq t\right]$$

(4) X $\leq_{PFLR4}$ Y if,

$$\min \{({}_{\alpha}\leq \beta \leq 1^{min}) f(\tilde{x})^{L}_{\alpha}, ({}_{\alpha}\leq \beta \leq 1^{min} g(\tilde{x})^{L}_{\alpha} \quad \leq$$

$$\min ({}_{\alpha}\leq \beta \leq 1\{\square^{min}) f\left[\lambda\tilde{y}^{L}_{\alpha} - \frac{t}{\lambda\tilde{y}^{L}_{\alpha}} \geq t\right], \quad ({}_{\alpha}\leq \beta \leq 1^{min} g\left[\lambda\tilde{y}^{L}_{\alpha} - \frac{t}{\lambda\tilde{y}^{L}_{\alpha}} \geq t\right]$$

And

$$\max \{({}_{\alpha}\leq \beta \leq 1^{max}) f(\tilde{x})^{L}_{\alpha}, ({}_{\alpha}\leq \beta \leq 1^{max} g(\tilde{x})^{L}_{\alpha} \quad \leq$$

$$\min ({}_{\alpha}\leq \beta \leq 1\{\square^{max}) f\left[\lambda\tilde{y}^{L}_{\alpha} - \frac{t}{\lambda\tilde{y}^{L}_{\alpha}} \geq t\right], \quad ({}_{\alpha}\leq \beta \leq 1^{max} g\left[\lambda\tilde{y}^{L}_{\alpha} - \frac{t}{\lambda\tilde{y}^{L}_{\alpha}} \geq t\right]$$

For all X ≤ Y, $\alpha \in [0,1]$.

3.4 Theorem: The two fuzzy random variables X and Y satisfy X $\leq_{PFLRi}$ Y if and only if X $\leq_{FLRi}$ α Y for all . (i=1,2,3,4).

Proof: Suppose that X $\leq_{PFLR2}$ Y. Thus we have that,

min

$$(_\alpha \leq \beta \leq 1\{ \square^{min}) f\left[(a)\tilde{y}_\alpha^L - \frac{t}{(a)\tilde{y}_\alpha^L} \geq t\right], \quad (_\alpha \leq \beta \leq 1^{min} g\left[(a\mu)\tilde{y}_\alpha^U - \frac{t}{(a)\tilde{y}_\alpha^U} \geq t\right]$$

And

max

$$(_\alpha \leq \beta \leq 1\{ \square^{max}) f\left[(a)\tilde{y}_\alpha^L - \frac{t}{(a)\tilde{y}_\alpha^L} \geq t\right], \quad (_\alpha \leq \beta \leq 1^{max} g\left[(a)\tilde{y}_\alpha^U - \frac{t}{(a)\tilde{y}_\alpha^U} \geq t\right]$$

is equal to

min

$$(_\alpha \leq \beta \leq 1\{ \square^{min}) f\left[(1/a)\tilde{y}_\alpha^L - \frac{t}{(1/a)\tilde{y}_\alpha^L} \geq t\right], \quad (_\alpha \leq \beta \leq 1^{min} g\left[(1/a)\tilde{y}_\alpha^U - \frac{t}{(1/a)\tilde{y}_\alpha^U} \geq t\right]$$

And

max

$$(_\alpha \leq \beta \leq 1\{ \square^{max}) f\left[(1/a)\tilde{y}_\alpha^L - \frac{t}{(1/a)\tilde{y}_\alpha^L} \geq t\right], \quad (_\alpha \leq \beta \leq 1^{max} g\left[(1/a)\tilde{y}_\alpha^U - \frac{t}{(1/a)\tilde{y}_\alpha^U} \geq t\right]$$

put( ) = λ, then equal to

$$\min (_\alpha \leq \beta \leq 1\{ \square^{min}) f\left[\lambda\tilde{y}_\alpha^L - \frac{t}{\lambda\tilde{y}_\alpha^L} \geq t\right], \quad (_\alpha \leq \beta \leq 1^{min} g\left[\lambda\tilde{y}_\alpha^U - \frac{t}{\lambda\tilde{y}_\alpha^U} \geq t\right] \text{ And}$$

$$\max (_\alpha \leq \beta \leq 1\{ \square^{max}) f\left[\lambda\tilde{y}_\alpha^L - \frac{t}{\lambda\tilde{y}_\alpha^L} \geq t\right], \quad (_\alpha \leq \beta \leq 1^{max} g\left[\lambda\tilde{y}_\alpha^U - \frac{t}{\lambda\tilde{y}_\alpha^U} \geq t\right]$$

since X $\leq_{PFLR2}$ Y, then greater than or equal to

$$\min (_\alpha \leq \beta \leq 1\{ \square^{min}) f\left[\lambda\tilde{x}_\alpha^L - \frac{t}{\lambda\tilde{x}_\alpha^L} \geq t\right], \quad (_\alpha \leq \beta \leq 1^{min} g\left[\lambda\tilde{x}_\alpha^U - \frac{t}{\lambda\tilde{x}_\alpha^U} \geq t\right] \text{ And}$$

$$\max (_\alpha \leq \beta \leq 1\{ \square^{max}) f\left[\lambda\tilde{x}_\alpha^L - \frac{t}{\lambda\tilde{x}_\alpha^L} \geq t\right], \quad (_\alpha \leq \beta \leq 1^{max} g\left[\lambda\tilde{x}_\alpha^U - \frac{t}{\lambda\tilde{x}_\alpha^U} \geq t\right]$$

equal to

$$\min \{(_\alpha \leq \beta \leq 1^{min}) f(a\tilde{x})_\alpha^L, (_\alpha \leq \beta \leq 1^{min} g(a\tilde{x})_\alpha^U \quad \text{And}$$

$$\max \{(_\alpha \leq \beta \leq 1^{max}) f(a\tilde{x})_\alpha^L, (_\alpha \leq \beta \leq 1^{max} g(a\tilde{x})_\alpha^U \quad .$$

Which implies that X $\leq_{PFLR2}$ Y. The similar proof are holds for the another ranking indexes.

4. General Reversed Fuzzy Hazard Rate Order

Let X be a non negative fuzzy random variable with density function $f(\tilde{x})$ and cumulative distribution function $\bar{F}(\tilde{x})$ respectively, and $\tilde{x}$ be a fuzzy random variables induced by X. The fuzzy function $\tilde{r}_{\tilde{F}}(\tilde{x})$ is a reversed Hazard rate of fuzzy random variables $\tilde{x}$, whenever its membership function is given by

$$\mu_{\tilde{r}}(y) = (_{\downarrow}0 \leq \alpha \leq 1^{\uparrow}sup)\alpha^{\uparrow}I\ \{r^{-}(x^{-})_{\downarrow}\alpha^{\uparrow}L\ ,\}(y)$$

Where, $\bar{F}(\tilde{x})_{\alpha}^{L}$ = min$\{\{(_{\downarrow}\alpha \leq \beta \leq 1^{\uparrow}min)G^{-}(x^{-}); x = x^{-}_{\downarrow}\beta^{\uparrow}L\ \}$, $\{(_{\alpha} \leq \beta \leq 1^{min})\bar{G}(\tilde{x}); x = \tilde{x}_{\beta}^{L}$ },

$\bar{F}(\tilde{x})_{\alpha}^{U}$ = max$\{\{(_{\downarrow}\alpha \leq \beta \leq 1^{\uparrow}max)G^{-}(x^{-}); x = x^{-}_{\downarrow}\beta^{\uparrow}L\ \}$, $\{(_{\alpha} \leq \beta \leq 1^{max})\bar{G}(\tilde{x}); x = \tilde{x}_{\beta}^{L}$ },

Such that the interval, $\bar{F}(\tilde{x})_{\alpha}^{L}$ and $\bar{F}(\tilde{x})_{\alpha}^{U}$ will contain all of the reversed Hazard rate of each $\tilde{x}_{\beta}^{L}$ and $\tilde{x}_{\beta}^{U}$ for $\beta \geq \alpha$.

4.1 Fuzzy Hazard Rate Order

Let X and Y be two non negative fuzzy random variables with continuous distribution functions and with Reversed Hazard rate functions $\tilde{r}(\tilde{x})$ and$(\tilde{x})$ respectively, then X is smaller than Y. We propose four relations

(1) X $\leq_{FRH1}$ Y if,

$$\frac{min\{(_{\alpha} \leq \beta \leq 1^{min})f\left(\tilde{x}_{\beta}^{L}\right), (_{\alpha} \leq \beta \leq 1^{min} f\left(\tilde{x}_{\beta}^{U}\right),}{min\{(_{\alpha} \leq \beta \leq 1^{min})\bar{F}\left(\tilde{x}_{\beta}^{L}\right)}, (_{\alpha} \leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}_{\beta}^{U}\right), \geq$$

$$\frac{min\{(_{\alpha} \leq \beta \leq 1^{min})g\left(\tilde{y}_{\beta}^{L}\right), (_{\alpha} \leq \beta \leq 1^{min} g\left(\tilde{y}_{\beta}^{U}\right),}{min\{(_{\alpha} \leq \beta \leq 1^{min})\bar{G}\left(\tilde{y}_{\beta}^{L}\right)}, (_{\alpha} \leq \beta \leq 1^{min}\bar{G}\left(\tilde{y}_{\beta}^{U}\right),$$

And

$$\frac{max\{(_{\alpha} \leq \beta \leq 1^{max}) f\left(\tilde{x}_{\beta}^{L}\right), (_{\alpha} \leq \beta \leq 1^{max} f\left(\tilde{x}_{\beta}^{U}\right),}{max\{(_{\alpha} \leq \beta \leq 1^{max}) \bar{F}\left(\tilde{x}_{\beta}^{L}\right)}, (_{\alpha} \leq \beta \leq 1^{max} \bar{F}\left(\tilde{x}_{\beta}^{U}\right). \geq$$

$$\frac{max\{(_{\alpha} \leq \beta \leq 1^{max}) g\left(\tilde{y}_{\beta}^{L}\right), (_{\alpha} \leq \beta \leq 1^{max} g\left(\tilde{y}_{\beta}^{U}\right),}{max\{(_{\alpha} \leq \beta \leq 1^{max}) \bar{G}\left(\tilde{y}_{\beta}^{L}\right)}, (_{\alpha} \leq \beta \leq 1^{max} \bar{G}\left(\tilde{y}_{\beta}^{U}\right).$$

(2) X $\leq_{FRH2}$ Y if,

$$\frac{min\{(_{\alpha} \leq \beta \leq 1^{min}) f\left(\tilde{x}_{\beta}^{L}\right), (_{\alpha} \leq \beta \leq 1^{min} f\left(\tilde{x}_{\beta}^{L}\right),}{min\{(_{\alpha} \leq \beta \leq 1^{min}) \bar{F}\left(\tilde{x}_{\beta}^{L}\right)}, (_{\alpha} \leq \beta \leq 1^{min} \bar{F}\left(\tilde{x}_{\beta}^{L}\right). \geq$$

$$\frac{min\{(_{\alpha} \leq \beta \leq 1^{min}) g\left(\tilde{y}_{\beta}^{U}\right), (_{\alpha} \leq \beta \leq 1^{min} g\left(\tilde{y}_{\beta}^{U}\right),}{min\{(_{\alpha} \leq \beta \leq 1^{min}) \bar{G}\left(\tilde{y}_{\beta}^{U}\right)}, (_{\alpha} \leq \beta \leq 1^{min} \bar{G}\left(y_{\beta}^{U}\right).$$

And

$$\frac{max\{(_{\alpha} \leq \beta \leq 1^{max}) f\left(\tilde{x}_{\beta}^{L}\right), (_{\alpha} \leq \beta \leq 1^{max} f\left(\tilde{x}_{\beta}^{L}\right),}{max\{(_{\alpha} \leq \beta \leq 1^{max}) \bar{F}\left(\tilde{x}_{\beta}^{L}\right)}, (_{\alpha} \leq \beta \leq 1^{max} \bar{F}\left(\tilde{x}_{\beta}^{L}\right). \geq$$

$$\frac{max\{(_{\alpha} \leq \beta \leq 1^{max}) g\left(\tilde{y}_{\beta}^{U}\right), (_{\alpha} \leq \beta \leq 1^{max} g\left(\tilde{y}_{\beta}^{U}\right),}{max\{(_{\alpha} \leq \beta \leq 1^{max}) \bar{G}\left(\tilde{y}_{\beta}^{U}\right)}, (_{\alpha} \leq \beta \leq 1^{max} \bar{G}\left(\tilde{y}_{\beta}^{U}\right).$$

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(3) X $\leq_{FRH3}$ Y if,

$$\frac{min\{(_{\alpha} \leq \beta \leq 1^{min}) f\left(\tilde{x}_{\beta}^{L}\right), (_{\alpha} \leq \beta \leq 1^{min} f\left(\tilde{x}_{\beta}^{L}\right),}{min\{(_{\alpha} \leq \beta \leq 1^{min}) \bar{F}\left(\tilde{x}_{\beta}^{L}\right)}, (_{\alpha} \leq \beta \leq 1^{min} \bar{F}\left(\tilde{x}_{\beta}^{L}\right). \geq$$

$$\frac{min\{(_{\alpha} \leq \beta \leq 1^{min}) g\left(\tilde{y}_{\beta}^{L}\right), (_{\alpha} \leq \beta \leq 1^{min} g\left(\tilde{y}_{\beta}^{L}\right),}{min\{(_{\alpha} \leq \beta \leq 1^{min}) \bar{G}\left(\tilde{y}_{\beta}^{L}\right)}, (_{\alpha} \leq \beta \leq 1^{min} \bar{G}\left(\tilde{y}_{\beta}^{L}\right).$$

And

$$\frac{max\{(_\alpha\leq \beta \leq 1^{max})f\left(\tilde{x}^L_\beta\right),(_\alpha\leq \beta \leq 1^{max}f\left(\tilde{x}^L_\beta\right),}{max\{(_\alpha\leq \beta \leq 1^{max})\bar{F}\left(\tilde{x}^L_\beta\right)},(_\alpha\leq \beta \leq 1^{max}\bar{F}\left(\tilde{x}^L_\beta\right), \geq$$

$$\frac{max\{(_\alpha\leq \beta \leq 1^{max})g\left(\tilde{y}^L_\beta\right),(_\alpha\leq \beta \leq 1^{max}g\left(\tilde{y}^L_\beta\right),}{max\{(_\alpha\leq \beta \leq 1^{max})\bar{G}\left(y^L_\beta\right)},(_\alpha\leq \beta \leq 1^{max}\bar{G}\left(y^L_\beta\right),$$

(4) X $\leq_{FRH4}$ Y if,

$$\frac{min\{(_\alpha\leq \beta \leq 1^{min})f\left(\tilde{x}^U_\beta\right),(_\alpha\leq \beta \leq 1^{min}f\left(\tilde{x}^U_\beta\right),}{min\{(_\alpha\leq \beta \leq 1^{min})\bar{F}\left(\tilde{x}^L_\beta\right)},(_\alpha\leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}^L_\beta\right), \geq$$

$$\frac{min\{(_\alpha\leq \beta \leq 1^{min})g\left(\tilde{y}^U_\beta\right),(_\alpha\leq \beta \leq 1^{min}g\left(\tilde{y}^U_\beta\right),}{min\{(_\alpha\leq \beta \leq 1^{min})\bar{G}\left(\tilde{y}^U_\beta\right)},(_\alpha\leq \beta \leq 1^{min}\bar{G}\left(\tilde{y}^U_\beta\right),$$

And

$$\frac{max\{(_\alpha\leq \beta \leq 1^{max})f\left(\tilde{x}^U_\beta\right),(_\alpha\leq \beta \leq 1^{max}f\left(\tilde{x}^U_\beta\right),}{max\{(_\alpha\leq \beta \leq 1^{max})\bar{F}\left(\tilde{x}^U_\beta\right)},(_\alpha\leq \beta \leq 1^{max}\bar{F}\left(\tilde{x}^U_\beta\right), \geq$$

$$\frac{max\{(_\alpha\leq \beta \leq 1^{max})g\left(y^U_\beta\right),(_\alpha\leq \beta \leq 1^{max}g\left(\tilde{y}^U_\beta\right),}{max\{(_\alpha\leq \beta \leq 1^{max})\bar{G}\left(\tilde{y}^U_\beta\right)},(_\alpha\leq \beta \leq 1^{max}\bar{G}\left(\tilde{y}^U_\beta\right),$$

For each α,β Є (0,1]∩Q, where $\overline{F}, f$ are the survival and density functions of X respectively and $\overline{G}, g$ are the survival and density functions of Y respectively.

4.2 Fuzzy Reversed Hazard Rate Order

Let X and Y be two non negative fuzzy random variables with continuous distribution functions and with Reversed Hazard rate functions $\tilde{r}(\tilde{x})$ and$(\tilde{x})$ respectively, then X is smaller than Y. We propose four relations

(1) X $\leq_{FRH1}$ Y if,

$$\frac{min\{(_\alpha \leq \beta \leq 1^{min}) f\left(\tilde{x}_\beta^L\right), (_\alpha \leq \beta \leq 1^{min} f\left(\tilde{x}_\beta^U\right),}{min\{(_\alpha \leq \beta \leq 1^{min}) \bar{F}\left(\tilde{x}_\beta^L\right)}, (_\alpha \leq \beta \leq 1^{min} \bar{F}\left(\tilde{x}_\beta^U\right), \leq$$

$$\frac{min\{(_\alpha \leq \beta \leq 1^{min}) g\left(\tilde{y}_\beta^L\right), (_\alpha \leq \beta \leq 1^{min} g\left(\tilde{y}_\beta^U\right),}{min\{(_\alpha \leq \beta \leq 1^{min}) \bar{G}\left(\tilde{y}_\beta^L\right)}, (_\alpha \leq \beta \leq 1^{min} \bar{G}\left(\tilde{y}_\beta^U\right),$$

And

$$\frac{max\{(_\alpha \leq \beta \leq 1^{max}) f\left(\tilde{x}_\beta^L\right), (_\alpha \leq \beta \leq 1^{max} f\left(\tilde{x}_\beta^U\right),}{max\{(_\alpha \leq \beta \leq 1^{max}) \bar{F}\left(\tilde{x}_\beta^L\right)}, (_\alpha \leq \beta \leq 1^{max} \bar{F}\left(\tilde{x}_\beta^U\right), \leq$$

$$\frac{max\{(_\alpha \leq \beta \leq 1^{max}) g\left(\tilde{y}_\beta^L\right), (_\alpha \leq \beta \leq 1^{max} g\left(\tilde{y}_\beta^U\right),}{max\{(_\alpha \leq \beta \leq 1^{max}) \bar{G}\left(y_\beta^L\right)}, (_\alpha \leq \beta \leq 1^{max} \bar{G}\left(\tilde{y}_\beta^U\right),$$

(2) X $\leq_{FRH2}$ Y if,

$$\frac{min\{(_\alpha \leq \beta \leq 1^{min}) f\left(\tilde{x}_\beta^L\right), (_\alpha \leq \beta \leq 1^{min} f\left(\tilde{x}_\beta^L\right),}{min\{(_\alpha \leq \beta \leq 1^{min}) \bar{F}\left(\tilde{x}_\beta^L\right)}, (_\alpha \leq \beta \leq 1^{min} \bar{F}\left(\tilde{x}_\beta^L\right), \leq$$

$$\frac{min\{(_\alpha \leq \beta \leq 1^{min}) g\left(y_\beta^U\right), (_\alpha \leq \beta \leq 1^{min} g\left(\tilde{y}_\beta^U\right),}{min\{(_\alpha \leq \beta \leq 1^{min}) \bar{G}\left(\tilde{y}_\beta^U\right)}, (_\alpha \leq \beta \leq 1^{min} \bar{G}\left(\tilde{y}_\beta^U\right),$$

And

$$\frac{max\{(_\alpha \leq \beta \leq 1^{max}) f\left(\tilde{x}_\beta^L\right), (_\alpha \leq \beta \leq 1^{max} f\left(\tilde{x}_\beta^L\right),}{max\{(_\alpha \leq \beta \leq 1^{max}) \bar{F}\left(\tilde{x}_\beta^L\right)}, (_\alpha \leq \beta \leq 1^{max} \bar{F}\left(\tilde{x}_\beta^L\right), \leq$$

$$\frac{max\{(_\alpha \leq \beta \leq 1^{max}) g\left(\tilde{y}_\beta^U\right), (_\alpha \leq \beta \leq 1^{max} g\left(\tilde{y}_\beta^U\right),}{max\{(_\alpha \leq \beta \leq 1^{max}) \bar{G}\left(\tilde{y}_\beta^U\right)}, (_\alpha \leq \beta \leq 1^{max} \bar{G}\left(\tilde{y}_\beta^U\right),$$

(3) X $\leq_{FRH3}$ Y if,

$$\frac{\min\{(_\alpha \le \beta \le 1^{\min})f\left(\tilde{x}^L_\beta\right), (_\alpha \le \beta \le 1^{\min} f\left(\tilde{x}^L_\beta\right),}{\min\{(_\alpha \le \beta \le 1^{\min})\bar{F}\left(\tilde{x}^L_\beta\right)}, (_\alpha \le \beta \le 1^{\min} \bar{F}\left(\tilde{x}^L_\beta\right), \le$$

$$\frac{\min\{(_\alpha \le \beta \le 1^{\min})g\left(\tilde{y}^L_\beta\right), (_\alpha \le \beta \le 1^{\min} g\left(\tilde{y}^L_\beta\right),}{\min\{(_\alpha \le \beta \le 1^{\min})\bar{G}\left(\tilde{y}^L_\beta\right)}, (_\alpha \le \beta \le 1^{\min} \bar{G}\left(\tilde{y}^L_\beta\right),$$

And

$$\frac{\max\{(_\alpha \le \beta \le 1^{\max})f\left(\tilde{x}^L_\beta\right), (_\alpha \le \beta \le 1^{\max} f\left(\tilde{x}^L_\beta\right),}{\max\{(_\alpha \le \beta \le 1^{\max})\bar{F}\left(\tilde{x}^L_\beta\right)}, (_\alpha \le \beta \le 1^{\max} \bar{F}\left(\tilde{x}^L_\beta\right), \le$$

$$\frac{\max\{(_\alpha \le \beta \le 1^{\max})g\left(\tilde{y}^L_\beta\right), (_\alpha \le \beta \le 1^{\max} g\left(\tilde{y}^L_\beta\right),}{\max\{(_\alpha \le \beta \le 1^{\max})\bar{G}\left(\tilde{y}^L_\beta\right)}, (_\alpha \le \beta \le 1^{\max} \bar{G}\left(\tilde{y}^L_\beta\right),$$

(4) X $\le_{FRH4}$ Y if,

$$\frac{\min\{(_\alpha \le \beta \le 1^{\min})f\left(\tilde{x}^U_\beta\right), (_\alpha \le \beta \le 1^{\min} f\left(\tilde{x}^U_\beta\right),}{\min\{(_\alpha \le \beta \le 1^{\min})\bar{F}\left(\tilde{x}^L_\beta\right)}, (_\alpha \le \beta \le 1^{\min} \bar{F}\left(\tilde{x}^L_\beta\right), \le$$

$$\frac{\min\{(_\alpha \le \beta \le 1^{\min})g\left(y^U_\beta\right), (_\alpha \le \beta \le 1^{\min} g\left(\tilde{y}^U_\beta\right),}{\min\{(_\alpha \le \beta \le 1^{\min})\bar{G}\left(\tilde{y}^U_\beta\right)}, (_\alpha \le \beta \le 1^{\min} \bar{G}\left(\tilde{y}^U_\beta\right),$$

And

$$\frac{\max\{(_\alpha \le \beta \le 1^{\max})f\left(\tilde{x}^U_\beta\right), (_\alpha \le \beta \le 1^{\max} f\left(\tilde{x}^U_\beta\right),}{\max\{(_\alpha \le \beta \le 1^{\max})\bar{F}\left(\tilde{x}^U_\beta\right)}, (_\alpha \le \beta \le 1^{\max} \bar{F}\left(\tilde{x}^U_\beta\right), \le$$

$$\frac{\max\{(_\alpha \le \beta \le 1^{\max})g\left(\tilde{y}^U_\beta\right), (_\alpha \le \beta \le 1^{\max} g\left(\tilde{y}^U_\beta\right),}{\max\{(_\alpha \le \beta \le 1^{\max})\bar{G}\left(\tilde{y}^U_\beta\right)}, (_\alpha \le \beta \le 1^{\max} \bar{G}\left(\tilde{y}^U_\beta\right),$$

For each α,β Є (0,1]∩Q, where $\overline{F}, f$ are the survival and density functions of X respectively and $\bar{G}, g$ are the survival and density functions of Y respectively.

4.3 Lemma: A continuous non negative fuzzy random variables X admits Up increasing fuzzy proportional Hazard rate order propose four property denoted by

(1) X$\in_{UIPHRO1}$ if,

$$\frac{min\{(_\alpha \leq \beta \leq 1^{min})\left[\frac{f(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right], (_\alpha \leq \beta \leq 1^{min}\left[\frac{f(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]}{min\{(_\alpha \leq \beta \leq 1^{min})\left[\frac{\bar{F}(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]}, (_\alpha \leq \beta \leq 1^{min}\left[\frac{\bar{F}(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]$$

$$\geq$$

$$\frac{min\{(_\alpha \leq \beta \leq 1^{min}) f\left(\tilde{x}_\beta^L\right), (_\alpha \leq \beta \leq 1^{min} f\left(\tilde{x}_\beta^U\right)}{min\{(_\alpha \leq \beta \leq 1^{min}) \bar{F}\left(\tilde{y}_\beta^L\right)}, (_\alpha \leq \beta \leq 1^{min} \bar{F}\left(\tilde{x}_\beta^U\right)$$

And

$$\frac{max\{(_\alpha \leq \beta \leq 1^{max})\left[\frac{f(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right], (_\alpha \leq \beta \leq 1^{max}\left[\frac{f(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]}{max\{(_\alpha \leq \beta \leq 1^{max})\left[\frac{\bar{F}(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]}, (_\alpha \leq \beta \leq 1^{max}\left[\frac{\bar{F}(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]$$

$$\geq$$

$$\frac{max\{(_\alpha \leq \beta \leq 1^{max}) f\left(\tilde{x}_\beta^L\right), (_\alpha \leq \beta \leq 1^{max} f\left(\tilde{x}_\beta^U\right)}{max\{(_\alpha \leq \beta \leq 1^{max}) \bar{F}\left(\tilde{x}_\beta^L\right)}, (_\alpha \leq \beta \leq 1^{max} F\left(\tilde{x}_\beta^U\right)$$

(2) X$\epsilon_{UIPHRO2}$ if,

$$\frac{min\{(_\alpha \leq \beta \leq 1^{min})\left[\frac{f(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right], (_\alpha \leq \beta \leq 1^{min}\left[\frac{f(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]}{min\{(_\alpha \leq \beta \leq 1^{min})\left[\frac{\bar{F}(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]}, (_\alpha \leq \beta \leq 1^{min}\left[\frac{\bar{F}(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]$$

$$\geq$$

$$\frac{min\{(_\alpha \leq \beta \leq 1^{min}) f\left(\tilde{x}_\beta^U\right), (_\alpha \leq \beta \leq 1^{min} f\left(\tilde{x}_\beta^U\right)}{min\{(_\alpha \leq \beta \leq 1^{min}) \bar{F}\left(\tilde{y}_\beta^U\right)}, (_\alpha \leq \beta \leq 1^{min} \bar{F}\left(\tilde{x}_\beta^U\right)$$

And

$$\frac{max\{(_\alpha \leq \beta \leq 1^{max})\left[\frac{f(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right], (_\alpha \leq \beta \leq 1^{max}\left[\frac{f(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]}{max\{(_\alpha \leq \beta \leq 1^{max})\left[\frac{\bar{F}(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]}, (_\alpha \leq \beta \leq 1^{max}\left[\frac{\bar{F}(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]$$

$$\geq$$

$$\frac{max\{(_\alpha \leq \beta \leq 1^{max}) f\left(\tilde{x}_\beta^U\right), (_\alpha \leq \beta \leq 1^{max} f\left(\tilde{x}_\beta^U\right)}{max\{(_\alpha \leq \beta \leq 1^{max}) \bar{F}\left(\tilde{x}_\beta^U\right)}, (_\alpha \leq \beta \leq 1^{max} F\left(\tilde{x}_\beta^U\right)$$

(3) $X \in_{UIPHRO3}$ if,

$$\frac{\min\{(_\alpha \leq \beta \leq 1^{min})\left[\frac{f(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right], (_\alpha \leq \beta \leq 1^{min}\left[\frac{f(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]}{\min\{(_\alpha \leq \beta \leq 1^{min})\left[\frac{\bar{F}(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]}, (_\alpha \leq \beta \leq 1^{min}\left[\frac{\bar{F}(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]$$

$$\geq$$

$$\frac{\min\{(_\alpha \leq \beta \leq 1^{min}) f\left(\tilde{x}_\beta^L\right), (_\alpha \leq \beta \leq 1^{min} f\left(\tilde{x}_\beta^L\right)}{\min\{(_\alpha \leq \beta \leq 1^{min})\bar{F}\left(\tilde{y}_\beta^L\right)}, (_\alpha \leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}_\beta^L\right)$$

And

$$\frac{\max\{(_\alpha \leq \beta \leq 1^{max})\left[\frac{f(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right], (_\alpha \leq \beta \leq 1^{max}\left[\frac{f(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]}{\max\{(_\alpha \leq \beta \leq 1^{max})\left[\frac{\bar{F}(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]}, (_\alpha \leq \beta \leq 1^{max}\left[\frac{\bar{F}(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]$$

$$\geq$$

$$\frac{\max\{(_\alpha \leq \beta \leq 1^{max}) f\left(\tilde{x}_\beta^L\right), (_\alpha \leq \beta \leq 1^{max} f\left(\tilde{x}_\beta^L\right)}{\max\{(_\alpha \leq \beta \leq 1^{max})\bar{F}\left(\tilde{x}_\beta^L\right)}, (_\alpha \leq \beta \leq 1^{max}\bar{F}\left(\tilde{x}_\beta^L\right)$$

(4) $X \in_{UIPHRO4}$ if,

$$\frac{\min\{(_\alpha \leq \beta \leq 1^{min})\left[\frac{f(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right], (_\alpha \leq \beta \leq 1^{min}\left[\frac{f(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]}{\min\{(_\alpha \leq \beta \leq 1^{min})\left[\frac{\bar{F}(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]}, (_\alpha \leq \beta \leq 1^{min}\left[\frac{\bar{F}(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]$$

$$\geq$$

$$\frac{\min\{(_\alpha \leq \beta \leq 1^{min}) f\left(\tilde{x}_\beta^U\right), (_\alpha \leq \beta \leq 1^{min} f\left(\tilde{x}_\beta^U\right)}{\min\{(_\alpha \leq \beta \leq 1^{min})\bar{F}\left(\tilde{y}_\beta^U\right)}, (_\alpha \leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}_\beta^U\right)$$

And

$$\frac{\max\{(_\alpha \leq \beta \leq 1^{max})\left[\frac{f(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right], (_\alpha \leq \beta \leq 1^{max}\left[\frac{f(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]}{\max\{(_\alpha \leq \beta \leq 1^{max})\left[\frac{\bar{F}(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]}, (_\alpha \leq \beta \leq 1^{max}\left[\frac{\bar{F}(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]$$

$$\geq$$

$$\frac{\min\{(_\alpha \leq \beta \leq 1^{min}) f\left(\tilde{x}_\beta^U\right), (_\alpha \leq \beta \leq 1^{min} f\left(\tilde{x}_\beta^U\right)}{\min\{(_\alpha \leq \beta \leq 1^{min})\bar{F}\left(\tilde{y}_\beta^U\right)}, (_\alpha \leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}_\beta^U\right)$$

The next theorem gives the relationship between the fuzzy Likelihood ratio and fuzzy Reversed Hazard rate orders.

4.4 Theorem: Suppose that X and Y are two non negative fuzzy random variables with fuzzy cumulative distributions functions $\bar{F}$ and $\bar{G}$, and also fuzzy Reversed Hazard rate order functions f and g respectively. The fuzzy Likelihood ratio ordering is stronger than the fuzzy Reversed Hazard rate ordering.

Proof: Suppose that X $\leq_{FLR1}$ Y. Then for all $\tilde{x} \leq \tilde{y}$, we can write

$$\min \{(_\alpha \leq \beta \leq 1^{min})f(\tilde{x})_\alpha^U, (_\alpha \leq \beta \leq 1^{min} g(\tilde{x})_\alpha^U \leq$$

$$\min \{(_\alpha \leq \beta \leq 1^{min})f(\tilde{y})_\alpha^U, (_\alpha \leq \beta \leq 1^{min} g(\tilde{y})_\alpha^U$$ And

$$\max \{(_\alpha \leq \beta \leq 1^{max})f(\tilde{x})_\alpha^U, (_\alpha \leq \beta \leq 1^{max} g(\tilde{x})_\alpha^U \leq$$

$$\max \{(_\alpha \leq \beta \leq 1^{max})f(\tilde{y})_\alpha^U, (_\alpha \leq \beta \leq 1^{max} g(\tilde{y})_\alpha^U$$

=

$$\frac{min\{(_\alpha \leq \beta \leq 1^{min})\bar{F}\left(\tilde{x}_\beta^U\right), (_\alpha \leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}_\beta^U\right),}{min\{(_\alpha \leq \beta \leq 1^{min})f\left(\tilde{x}_\beta^U\right)}, (_\alpha \leq \beta \leq 1^{min} f\left(\tilde{x}_\beta^U\right). \geq$$

$$\frac{min\{(_\alpha \leq \beta \leq 1^{min})\bar{G}\left(\tilde{y}_\beta^U\right), (_\alpha \leq \beta \leq 1^{min}\bar{G}\left(\tilde{y}_\beta^U\right),}{min\{(_\alpha \leq \beta \leq 1^{min})g\left(\tilde{y}_\beta^U\right)}, (_\alpha \leq \beta \leq 1^{min} g\left(\tilde{y}_\beta^U\right).$$

And

$$\frac{max\{(_\alpha \leq \beta \leq 1^{max})\bar{F}\left(\tilde{x}_\beta^U\right), (_\alpha \leq \beta \leq 1^{max}\bar{F}\left(\tilde{x}_\beta^U\right),}{max\{(_\alpha \leq \beta \leq 1^{max})f\left(\tilde{x}_\beta^U\right)}, (_\alpha \leq \beta \leq 1^{max}\left(\tilde{x}_\beta^U\right). \geq$$

$$\frac{max\{(_\alpha \leq \beta \leq 1^{max})\bar{G}\left(\tilde{y}_\beta^U\right), (_\alpha \leq \beta \leq 1^{max}\bar{G}\left(\tilde{y}_\beta^U\right),}{max\{(_\alpha \leq \beta \leq 1^{max})g\left(\tilde{y}_\beta^U\right)}, (_\alpha \leq \beta \leq 1^{max} g\left(\tilde{y}_\beta^U\right).$$

=

$\int_0^y \equiv (min\{(_\alpha \leq \beta \leq 1^{min}) \bar{F}(\tilde{x}_\beta^U), (_\alpha \leq \beta \leq 1^{min}) \bar{F}(\tilde{x}_\beta^U), \quad \}/min\{(_\alpha \leq$

$\geq$

]dx

By definition* $\geq$

$\int_{\downarrow 0}^{\uparrow} y \equiv (min\{(\downarrow\alpha \le \beta \le 1^{\uparrow} min) F^{-}(x^{-}{}_{\downarrow}\beta^{\uparrow}L), (\downarrow\alpha \le \beta \le 1^{\uparrow} min) F^{-}(x^{-}{}_{\downarrow}\beta^{\uparrow}L), \quad \}/min\{(\downarrow\alpha \le$

$\ge$]dx

Is equal to

$$\min \{(_\alpha \le \beta \le 1^{min}) f(\tilde{x})_\alpha^L, (_\alpha \le \beta \le 1^{min} g(\tilde{x})_\alpha^L \le$$

$$\min \{(_\alpha \le \beta \le 1^{min}) f(\tilde{y})_\alpha^L, (_\alpha \le \beta \le 1^{min} g(\tilde{y})_\alpha^L$$ And

$$\max \{(_\alpha \le \beta \le 1^{max}) f(\tilde{x})_\alpha^L, (_\alpha \le \beta \le 1^{max} g(\tilde{x})_\alpha^L \le$$

$$\max \{(_\alpha \le \beta \le 1^{max}) f(\tilde{y})_\alpha^L, (_\alpha \le \beta \le 1^{max} g(\tilde{y})_\alpha^L$$

Now regarding to Definition** proof is complete.

Suppose that $X_1, X_2, X_3, \ldots, X_n$ and $Y_1, Y_2, Y_3, \ldots, Y_n$ are two independent fuzzy random samples of size n.

Induced by $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \ldots, \tilde{x}_n$ (with cumulative distribution functions F) and $\tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \ldots, \tilde{y}_n$ (with cumulative distribution functions G), respectively. We denote $\tilde{r}_{\tilde{F}_k}(\tilde{x})$ as the fuzzy reversed Hazard rate of the k-th fuzzy statistic $X_{k:n}$, as the following,

$$\mu_{\tilde{r}_{\tilde{F}_k}(\tilde{x})}(y) = \{(_\downarrow\alpha \le \beta \le 1^{\uparrow} max) \square r^{-}{}_{\downarrow}(F^{-}{}_{\downarrow}k)(x^{-})\square_{\downarrow}\alpha^{\uparrow}L \quad ,$$

$$(_\alpha \le \beta \le 1^{max} \tilde{r}_{\tilde{F}_k}(\tilde{x})_\alpha^U \}(y)$$

Where,

$$= \min \{(_\alpha \le \beta \le 1^{min}) \left[\frac{\left(n!/(k-1)!(n-k)! \right) \left(\frac{f(x)}{F(x)} \right)}{\sum_k^n \frac{n!}{j!(n-j)!} \left[\frac{F(x)}{f(x)} \right] (k-j)} \right] : x = \tilde{x}_\beta^L \ ,$$

$$\min \{(_\alpha \le \beta \le 1^{min}) \left[\frac{\left(n!/(k-1)!(n-k)! \right) \left(\frac{f(x)}{F(x)} \right)}{\sum_k^n \frac{n!}{j!(n-j)!} \left[\frac{F(x)}{f(x)} \right] (k-j)} \right] : x = \tilde{x}_\beta^U$$

And

$$\tilde{r}_{\tilde{F}_k}(\tilde{x})_\alpha^U = \max \{(_\alpha \le \beta \le 1^{min}) \left[\frac{\left(n!/(k-1)!(n-k)! \right) \left(\frac{f(x)}{F(x)} \right)}{\sum_k^n \frac{n!}{j!(n-j)!} \left[\frac{F(x)}{f(x)} \right] (k-j)} \right] : x = \tilde{x}_\beta^L \ ,$$

$$\max \{(_\alpha \le \beta \le 1^{min}) \left[\frac{\left(n!/(k-1)!(n-k)! \right) \left(\frac{f(x)}{F(x)} \right)}{\sum_k^n \frac{n!}{j!(n-j)!} \left[\frac{F(x)}{f(x)} \right] (k-j)} \right] : x = \tilde{x}_\beta^U$$

4.5 Theorem: Suppose that $X_i \le_{FRH1} Y_i$ for all $i = 1,2,3,\ldots,n$ and

$$\left\{ \frac{min\{(_{\alpha}\leq \beta \leq 1^{min}) f\left(\tilde{x}_{\beta}^{L}\right), (_{\alpha}\leq \beta \leq 1^{min} f\left(\tilde{x}_{\beta}^{U}\right),}{min\{(_{\alpha}\leq \beta \leq 1^{min})\bar{F}\left(\tilde{x}_{\beta}^{L}\right)}, (_{\alpha}\leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}_{\beta}^{U}\right), \leq \frac{min\{(_{\alpha}\leq \beta \leq 1^{min}) g\left(\tilde{y}_{\beta}^{L}\right), (_{\alpha}\leq \beta \leq 1^{min} g\left(\tilde{y}_{\beta}^{U}\right),}{min\{(_{\alpha}\leq \beta \leq 1^{min})\bar{G}\left(\tilde{y}_{\beta}^{L}\right)}, (_{\alpha}\leq \beta \leq 1^{min}\bar{G}\left(\tilde{y}_{\beta}^{U}\right), \right\} \geq$$

$$\left\{ \frac{max\{(_{\alpha}\leq \beta \leq 1^{max}) f\left(\tilde{x}_{\beta}^{L}\right), (_{\alpha}\leq \beta \leq 1^{max} f\left(\tilde{x}_{\beta}^{U}\right),}{max\{(_{\alpha}\leq \beta \leq 1^{max})\bar{F}\left(\tilde{x}_{\beta}^{L}\right)}, (_{\alpha}\leq \beta \leq 1^{max}\bar{F}\left(\tilde{x}_{\beta}^{U}\right), \leq \frac{max\{(_{\alpha}\leq \beta \leq 1^{max}) g\left(\tilde{y}_{\beta}^{L}\right), (_{\alpha}\leq \beta \leq 1^{max} g\left(\tilde{y}_{\beta}^{U}\right),}{max\{(_{\alpha}\leq \beta \leq 1^{max})\bar{G}\left(y_{\beta}^{L}\right)}, (_{\alpha}\leq \beta \leq 1^{max}\bar{G}\left(\tilde{y}_{\beta}^{U}\right), \right\}$$

Then $X_{i:n} \leq_{FRH1} Y_{i:n}$.

Proof: We can prove easily that the function $\left[\sum_{k}^{n} \frac{n!}{j!\,(n-j)!}\left[\frac{1-\tilde{x}}{\tilde{x}}\right](k-j)\right]^{-1}$ is non decreasing function in x .
Since that $X_i \leq_{FRHR1} Y_i$, we can conclude that

$$\frac{1}{\left[\frac{\sum_{k}^{n} \frac{n!}{j!\,(n-j)!}\left[\left(1-F\left(\tilde{x}\right)_{\alpha}^{L}\right)\right)}{f\left(\tilde{x}\right)_{\alpha}^{L}}\right](k-j]} \geq \frac{1}{\left[\frac{\sum_{k}^{n} \frac{n!}{j!\,(n-j)!}\left[\left(1-G\left(\tilde{x}\right)_{\alpha}^{L}\right)\right)}{g\left(\tilde{x}\right)_{\alpha}^{L}}\right](k-j]}$$

Let us suppose that

$$A=\left\{ \frac{\frac{n!}{j!\,(n-1)!}\, min\{(_{\alpha}\leq \beta \leq 1^{min})f\left(\tilde{x}^{L}_{\beta}\right),(_{\alpha}\leq \beta \leq 1^{min} f\left(\tilde{x}^{U}_{\beta}\right),}{min\{(_{\alpha}\leq \beta \leq 1^{min})\bar{F}\left(\tilde{x}^{L}_{\beta}\right)},(_{\alpha}\leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}^{U}_{\beta}\right), \right\}$$

$$B=\left\{ \frac{\frac{n!}{j!\,(n-1)!}\, min\{(_{\alpha}\leq \beta \leq 1^{min})g\left(\tilde{x}^{L}_{\beta}\right),(_{\alpha}\leq \beta \leq 1^{min} g\left(\tilde{x}^{U}_{\beta}\right),}{min\{(_{\alpha}\leq \beta \leq 1^{min})\bar{G}\left(\tilde{x}^{L}_{\beta}\right)},(_{\alpha}\leq \beta \leq 1^{min}\bar{G}\left(\tilde{x}^{U}_{\beta}\right), \right\}$$

$$C=\left\{ \frac{\frac{n!}{j!\,(n-1)!}\, max\{(_{\alpha}\leq \beta \leq 1^{max})g\left(\tilde{x}^{L}_{\beta}\right),(_{\alpha}\leq \beta \leq 1^{max} g\left(\tilde{x}^{U}_{\beta}\right),}{max\{(_{\alpha}\leq \beta \leq 1^{max})\bar{G}\left(\tilde{x}^{L}_{\beta}\right)},(_{\alpha}\leq \beta \leq 1^{max}\bar{G}\left(\tilde{x}^{U}_{\beta}\right), \right\}$$

$$D=\left\{ \frac{\frac{n!}{j!\,(n-1)!}\, max\{(_{\alpha}\leq \beta \leq 1^{max})f\left(\tilde{x}^{L}_{\beta}\right),(_{\alpha}\leq \beta \leq 1^{max} f\left(\tilde{x}^{U}_{\beta}\right),}{max\{(_{\alpha}\leq \beta \leq 1^{max})\bar{F}\left(\tilde{x}^{L}_{\beta}\right)},(_{\alpha}\leq \beta \leq 1^{max}\bar{F}\left(\tilde{x}^{U}_{\beta}\right), \right\}$$

Now by using the minimum and maximum property and inequality 3,we have min $\{(_\alpha \le \beta \le 1^{min})\left[\dfrac{\left(n!/(k-1)!\,(n-k)!\right)\left(\dfrac{f(x)}{F(x)}\right)}{\sum_k^n \dfrac{n!}{j!\,(n-j)!}\left[\dfrac{F(x)}{f(x)}\right](k-j)}\right]$: x = $\tilde{x}_\beta^L$ ≥

max $\{(_\alpha \le \beta \le 1^{min})\left[\dfrac{\left(n!/(k-1)!\,(n-k)!\right)\left(\dfrac{f(x)}{F(x)}\right)}{\sum_k^n \dfrac{n!}{j!\,(n-j)!}\left[\dfrac{F(x)}{f(x)}\right](k-j)}\right]$: x = $\tilde{x}_\beta^U$

or equivalently ≥ $\tilde{r}_{\tilde{F}_k}(\tilde{x})_\alpha^U$

And hence Then $X_{i:n} \le_{FRH1} Y_{i:n}$

5. Fuzzy Mean Inactivity Time Order

Let X be a non negative fuzzy random variable with density function $f(\tilde{t})$ and cumulative distribution function $\bar{F}(\tilde{t})$ respectively, and $\tilde{x}$ be a fuzzy random variables induced by X. The fuzzy function $\tilde{m}_{\tilde{F}}(t\ )$ is said a fuzzy Mean inactivity time of fuzzy random variables $\tilde{x}$, whenever its membership function is given by,

$$\mu_{\tilde{m}}(y) = (_\downarrow 0 \le \alpha \le 1^\uparrow sup)\alpha^\uparrow I\ \{m^{\sim}\square(t^{\sim})\square_\downarrow \alpha^\uparrow L\ ,\}(y)$$

Where, =

min$\{\{(_\downarrow\alpha \le \beta \le 1^\uparrow min)m(t^{\sim}); x = t^{\sim}{}_\downarrow\beta^\uparrow L\ \}$, $\{(_\alpha \le \beta \le 1^{min})m(\tilde{t}); x = \tilde{t}_\beta^L$ },

$$\int_0^t \tilde{m}(\tilde{t})_\alpha^U dx\ =$$

max$\{\{(_\downarrow\alpha \le \beta \le 1^\uparrow max)m(t^{\sim}); x = t^{\sim}{}_\downarrow\beta^\uparrow L\ \}$, $\{(_\alpha \le \beta \le 1^{max})m(\tilde{t}); x = \tilde{t}_\beta^L$ },

Such that the interval, $\tilde{m}(\tilde{t})_\alpha^L$ and $\tilde{m}(\tilde{t})_\alpha^U$ will contain all of the Mean inactivity time of each $\tilde{t}_\beta^L$ and $\tilde{t}_\beta^U$ for β ≥ α.

5.1 Definition: Let X and Y are two non negative fuzzy random variables with continuous distribution functions an fuzzy Mean inactivity time with functions ($\tilde{x}$) and ($\tilde{y}$) respectively, then X is smaller than Y. We propose four relations

(1)X $\leq_{FMIT1}$ Y if,

$$\frac{\left(\int_0^t min\{({}_\alpha \leq \beta \leq 1^{min})F(\tilde{x})_\beta^L, ({}_\alpha \leq \beta \leq 1^{min}\right) F(\tilde{x})_\beta^U\, dx}{min\{({}_\alpha \leq \beta \leq 1^{min})F(\tilde{t})_\beta^L}, ({}_\alpha \leq \beta \leq 1^{min} F(t)_\beta^U$$

$$\leq \frac{\left(\int_0^t min\{({}_\alpha \leq \beta \leq 1^{min})G(\tilde{y})_\beta^L, ({}_\alpha \leq \beta \leq 1^{min}\right) G(\tilde{y})_\beta^U\, dx}{min\{({}_\alpha \leq \beta \leq 1^{min})G(\tilde{t})_\beta^L}, ({}_\alpha \leq \beta \leq 1^{min} G(t)_\beta^U$$

And

$$\frac{\left(\int_0^t max\{({}_\alpha \leq \beta \leq 1^{max})F(\tilde{x})_\beta^L, ({}_\alpha \leq \beta \leq 1^{max}\right) F(\tilde{x})_\beta^U\, dx}{max\{({}_\alpha \leq \beta \leq 1^{max})F(\tilde{t})_\beta^L}, ({}_\alpha \leq \beta \leq 1^{max} F(t)_\beta^U$$

$$\leq \frac{\left(\int_0^t max\{({}_\alpha \leq \beta \leq 1^{max})G(\tilde{y})_\beta^L, ({}_\alpha \leq \beta \leq 1^{max}\right) G(\tilde{y})_\beta^U\, dx}{max\{({}_\alpha \leq \beta \leq 1^{max})G(\tilde{t})_\beta^L}, ({}_\alpha \leq \beta \leq 1^{max} G(t)_\beta^U$$

(2)X $\leq_{FMIT2}$ Y if,

$$\frac{\left(\int_0^t min\{({}_\alpha \leq \beta \leq 1^{min})F(\tilde{x})_\beta^L, ({}_\alpha \leq \beta \leq 1^{min}\right) F(\tilde{x})_\beta^L\, dx}{min\{({}_\alpha \leq \beta \leq 1^{min})F(\tilde{t})_\beta^L}, ({}_\alpha \leq \beta \leq 1^{min} F(t)_\beta^L$$

$$\leq \frac{\left(\int_0^t min\{({}_\alpha \leq \beta \leq 1^{min})G(\tilde{y})_\beta^U, ({}_\alpha \leq \beta \leq 1^{min}\right) G(\tilde{y})_\beta^U\, dx}{min\{({}_\alpha \leq \beta \leq 1^{min})G(\tilde{t})_\beta^U}, ({}_\alpha \leq \beta \leq 1^{min} G(t)_\beta^U$$

And

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$$\frac{\left(\int_0^t max\{({}_{\alpha}\leq \beta \leq 1^{max})F(\tilde{x})_{\beta}^{L}, ({}_{\alpha}\leq \beta \leq 1^{max}\right) F(\tilde{x})_{\beta}^{L}\, dx}{max\{({}_{\alpha}\leq \beta \leq 1^{max})F(\tilde{t})_{\beta}^{L}}, ({}_{\alpha}\leq \beta \leq 1^{max} F(\tilde{t})_{\beta}^{L}$$

$$\leq \frac{\left(\int_0^t max\{({}_{\alpha}\leq \beta \leq 1^{max})G(\tilde{y})_{\beta}^{U}, ({}_{\alpha}\leq \beta \leq 1^{max}\right) G(\tilde{y})_{\beta}^{U}\, dx}{max\{({}_{\alpha}\leq \beta \leq 1^{max})G(\tilde{t})_{\beta}^{U}}, ({}_{\alpha}\leq \beta \leq 1^{max} G(\tilde{t})_{\beta}^{U}$$

(3) X $\leq_{FMIT3}$ Y if,

$$\frac{\left(\int_0^t min\{({}_{\alpha}\leq \beta \leq 1^{min})F(\tilde{x})_{\beta}^{L}, ({}_{\alpha}\leq \beta \leq 1^{min}\right) F(\tilde{x})_{\beta}^{L}\, dx}{min\{({}_{\alpha}\leq \beta \leq 1^{min})F(\tilde{t})_{\beta}^{L}}, ({}_{\alpha}\leq \beta \leq 1^{min} F(\tilde{t})_{\beta}^{L}$$

$$\leq \frac{\left(\int_0^t min\{({}_{\alpha}\leq \beta \leq 1^{min})G(\tilde{y})_{\beta}^{L}, ({}_{\alpha}\leq \beta \leq 1^{min}\right) G(\tilde{y})_{\beta}^{L}\, dx}{min\{({}_{\alpha}\leq \beta \leq 1^{min})G(\tilde{t})_{\beta}^{L}}, ({}_{\alpha}\leq \beta \leq 1^{min} G(\tilde{t})_{\beta}^{L}$$

And

$$\frac{\left(\int_0^t max\{({}_{\alpha}\leq \beta \leq 1^{max})F(\tilde{x})_{\beta}^{L}, ({}_{\alpha}\leq \beta \leq 1^{max}\right) F(\tilde{x})_{\beta}^{L}\, dx}{max\{({}_{\alpha}\leq \beta \leq 1^{max})F(\tilde{t})_{\beta}^{L}}, ({}_{\alpha}\leq \beta \leq 1^{max} F(\tilde{t})_{\beta}^{L}$$

$$\leq \frac{\left(\int_0^t max\{({}_{\alpha}\leq \beta \leq 1^{max})G(\tilde{y})_{\beta}^{L}, ({}_{\alpha}\leq \beta \leq 1^{max}\right) G(\tilde{y})_{\beta}^{L}\, dx}{max\{({}_{\alpha}\leq \beta \leq 1^{max})G(\tilde{t})_{\beta}^{L}}, ({}_{\alpha}\leq \beta \leq 1^{max} G(\tilde{t})_{\beta}^{L}$$

(4) X $\leq_{FMIT3}$ Y if,

$$\frac{\left(\int_0^t min\{({}_{\alpha}\leq \beta \leq 1^{min})F(\tilde{x})_{\beta}^{U}, ({}_{\alpha}\leq \beta \leq 1^{min}\right) F(\tilde{x})_{\beta}^{U}\, dx}{min\{({}_{\alpha}\leq \beta \leq 1^{min})F(\tilde{t})_{\beta}^{U}}, ({}_{\alpha}\leq \beta \leq 1^{min} F(\tilde{t})_{\beta}^{U}$$

$$\leq \frac{\left(\int_0^t min\{({}_{\alpha}\leq \beta \leq 1^{min})G(\tilde{y})_{\beta}^{U}, ({}_{\alpha}\leq \beta \leq 1^{min}\right) G(\tilde{y})_{\beta}^{U}\, dx}{min\{({}_{\alpha}\leq \beta \leq 1^{min})G(\tilde{t})_{\beta}^{U}}, ({}_{\alpha}\leq \beta \leq 1^{min} G(\tilde{t})_{\beta}^{U}$$

And

$$\frac{\left(\int_0^t max\{({}_{\alpha}\leq \beta \leq 1^{max})F(\tilde{x})_{\beta}^{U}, ({}_{\alpha}\leq \beta \leq 1^{max}\right) F(\tilde{x})_{\beta}^{U}\, dx}{max\{({}_{\alpha}\leq \beta \leq 1^{max})F(\tilde{t})_{\beta}^{U}}, ({}_{\alpha}\leq \beta \leq 1^{max} F(\tilde{t})_{\beta}^{U}$$

$$\leq \frac{\left(\int_0^t max\{(_\alpha \leq \beta \leq 1^{max})F(\tilde{y})_\beta^U, (_\alpha \leq \beta \leq 1^{max}\right) F(\tilde{y})_\beta^U \, dx}{max\{(_\alpha \leq \beta \leq 1^{max})G(\tilde{t})_\beta^U}, (_\alpha \leq \beta \leq 1^{max} G(\tilde{t})_\beta^U$$

For each α,β Є (0,1]∩Q, where F, G are the survival and density functions of X and Y respectively.

5.2 Lemma (Decreasing Mean Inactivity)

Suppose that X and Y are two fuzzy random variables with fuzzy cumulative distribution functions F($\tilde{x}$) and G($\tilde{y}$) respectively then, we propose four relations

(1) X $\leq_{FMIT1}$ Y if,

$$\frac{\left(\int_0^t min\{(_\alpha \leq \beta \leq 1^{min})F(\tilde{x})_\beta^L, (_\alpha \leq \beta \leq 1^{min}\right) F(\tilde{x})_\beta^U \, dx}{min\{(_\alpha \leq \beta \leq 1^{min})F(\tilde{t})_\beta^L}, (_\alpha \leq \beta \leq 1^{min} F(\tilde{t})_\beta^U$$

$$\leq \frac{\left(\int_0^t min\{(_\alpha \leq \beta \leq 1^{min})G(\tilde{y}-t)_\beta^L, (_\alpha \leq \beta \leq 1^{min}\right) G(\tilde{y}-t)_\beta^U \, dx}{min\{(_\alpha \leq \beta \leq 1^{min})G(\tilde{t})_\beta^L}, (_\alpha \leq \beta \leq 1^{min} G(\tilde{t})_\beta^U$$

And

$$\frac{\left(\int_0^t max\{(_\alpha \leq \beta \leq 1^{max})F(\tilde{x})_\beta^L, (_\alpha \leq \beta \leq 1^{max}\right) F(\tilde{x})_\beta^U \, dx}{max\{(_\alpha \leq \beta \leq 1^{max})F(\tilde{t})_\beta^L}, (_\alpha \leq \beta \leq 1^{max} F(\tilde{t})_\beta^U$$

$$\leq \frac{\left(\int_0^t max\{(_\alpha \leq \beta \leq 1^{max})G(\tilde{y}-t)_\beta^L, (_\alpha \leq \beta \leq 1^{max}\right) G(\tilde{y}-t)_\beta^U \, dx}{max\{(_\alpha \leq \beta \leq 1^{max})G(\tilde{t})_\beta^L}, (_\alpha \leq \beta \leq 1^{max} G(\tilde{t})_\beta^U$$

(2) X $\leq_{FMIT2}$ Y if,

$$\frac{\left(\int_0^t min\{(_\alpha \leq \beta \leq 1^{min})F(\tilde{x})_\beta^L, (_\alpha \leq \beta \leq 1^{min}\right) F(\tilde{x})_\beta^L \, dx}{min\{(_\alpha \leq \beta \leq 1^{min})F(\tilde{t})_\beta^L}, (_\alpha \leq \beta \leq 1^{min} F(\tilde{t})_\beta^L$$

$$\leq \frac{\left(\int_0^t min\{(_\alpha \leq \beta \leq 1^{min})G(\tilde{y}-t)_\beta^U, (_\alpha \leq \beta \leq 1^{min}\right) G(\tilde{y})_\beta^U \, dx}{min\{(_\alpha \leq \beta \leq 1^{min})G(\tilde{t})_\beta^U}, (_\alpha \leq \beta \leq 1^{min} G(\tilde{t})_\beta^U$$

And

$$\frac{\left(\int_0^t max\{(_\alpha\leq \beta \leq 1^{max})F(\tilde{x})_\beta^L,(_\alpha\leq \beta \leq 1^{max}\right)F(\tilde{x})_\beta^L\,dx}{max\{(_\alpha\leq \beta \leq 1^{max})F(\tilde{t})_\beta^L},(_\alpha\leq \beta \leq 1^{max}F(\tilde{t})_\beta^L$$
$$\leq \frac{\left(\int_0^t max\{(_\alpha\leq \beta \leq 1^{max})G(\tilde{y}-t)_\beta^U,(_\alpha\leq \beta \leq 1^{max}\right)G(\tilde{y}-t)_\beta^U\,dx}{max\{(_\alpha\leq \beta \leq 1^{max})G(\tilde{t})_\beta^U},(_\alpha\leq \beta \leq 1^{max}G(\tilde{t})_\beta^U$$

(3) X $\leq_{FMIT3}$ Y if,

$$\frac{\left(\int_0^t min\{(_\alpha\leq \beta \leq 1^{min})F(\tilde{x})_\beta^L,(_\alpha\leq \beta \leq 1^{min}\right)F(\tilde{x})_\beta^L\,dx}{min\{(_\alpha\leq \beta \leq 1^{min})F(\tilde{t})_\beta^L},(_\alpha\leq \beta \leq 1^{min}F(\tilde{t})_\beta^L$$

$$\leq \frac{\left(\int_0^t min\{(_\alpha\leq \beta \leq 1^{min})G(\tilde{y}-t)_\beta^L,(_\alpha\leq \beta \leq 1^{min}\right)G(\tilde{y}-t)_\beta^L\,dx}{min\{(_\alpha\leq \beta \leq 1^{min})G(\tilde{t})_\beta^L},(_\alpha\leq \beta \leq 1^{min}G(\tilde{t})_\beta^L$$

And

$$\frac{\left(\int_0^t max\{(_\alpha\leq \beta \leq 1^{max})F(\tilde{x})_\beta^L,(_\alpha\leq \beta \leq 1^{max}\right)F(\tilde{x})_\beta^L\,dx}{max\{(_\alpha\leq \beta \leq 1^{max})F(\tilde{t})_\beta^L},(_\alpha\leq \beta \leq 1^{max}F(\tilde{t})_\beta^L$$
$$\leq \frac{\left(\int_0^t max\{(_\alpha\leq \beta \leq 1^{max})G(\tilde{y}-t)_\beta^L,(_\alpha\leq \beta \leq 1^{max}\right)G(\tilde{y}-t)_\beta^L\,dx}{max\{(_\alpha\leq \beta \leq 1^{max})G(\tilde{t})_\beta^L},(_\alpha\leq \beta \leq 1^{max}G(\tilde{t})_\beta^L$$

(4) X $\leq_{FMIT4}$ Y if,

$$\frac{\left(\int_0^t min\{(_\alpha\leq \beta \leq 1^{min})F(\tilde{x})_\beta^U,(_\alpha\leq \beta \leq 1^{min}\right)F(\tilde{x})_\beta^U\,dx}{min\{(_\alpha\leq \beta \leq 1^{min})F(\tilde{t})_\beta^U},(_\alpha\leq \beta \leq 1^{min}F(\tilde{t})_\beta^U$$

$$\leq \frac{\left(\int_0^t min\{(_\alpha\leq \beta \leq 1^{min})G(\tilde{y}-t)_\beta^U,(_\alpha\leq \beta \leq 1^{min}\right)G(\tilde{y}-t)_\beta^U\,dx}{min\{(_\alpha\leq \beta \leq 1^{min})G(\tilde{t})_\beta^U},(_\alpha\leq \beta \leq 1^{min}G(\tilde{t})_\beta^U$$

And

$$\frac{\left(\int_0^t max\{(_\alpha\leq \beta \leq 1^{max})F(\tilde{x})_\beta^U, (_\alpha\leq \beta \leq 1^{max}\right)F(\tilde{x})_\beta^U\, dx}{max\{(_\alpha\leq \beta \leq 1^{max})F(\tilde{t})_\beta^U}, (_\alpha\leq \beta \leq 1^{max} F(\tilde{t})_\beta^U$$

$$\leq \frac{\left(\int_0^t max\{(_\alpha\leq \beta \leq 1^{max})F(\tilde{y}-t)_\beta^U, (_\alpha\leq \beta \leq 1^{max}\right)F(\tilde{y}-t)_\beta^U\, dx}{max\{(_\alpha\leq \beta \leq 1^{max})G(\tilde{t})_\beta^U}, (_\alpha\leq \beta \leq 1^{max} G(\tilde{t})_\beta^U$$

Is decreasing in $x > 0$.

The following theorem gives the relationship between the fuzzy mean inactivity time and fuzzy reversed hazard rate order.

5.3 Theorem: Let X and Y are two fuzzy random variables with fuzzy cumulative distribution functions F and G respectively

(a) If X $\leq_{FRHR1}$ Y, then X $\leq_{FMIT2}$ Y.

(b) If X $\leq_{FRHR1}$ Y, then X $\leq_{FMIT3}$ Y.

(c) If X $\leq_{FRHR1}$ Y, then X $\leq_{FMIT4}$ Y.

(d) If X $\leq_{FRHR1}$ Y, then X $\leq_{FMIT1}$ Y.

Proof: Suppose that X $\leq_{FRHR1}$ Y. based on lemma,

$$\Rightarrow \frac{min\{(_\alpha\leq \beta \leq 1^{min})\left[\frac{f(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right], (_\alpha\leq \beta \leq 1^{min}\left[\frac{f(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]}{min\{(_\alpha\leq \beta \leq 1^{min})\left[\frac{\bar{F}(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]}, (_\alpha\leq \beta \leq 1^{min}\left[\frac{\bar{F}(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]$$

And

$$\frac{max\{(_\alpha\leq \beta \leq 1^{max})\left[\frac{f(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right], (_\alpha\leq \beta \leq 1^{max}\left[\frac{f(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]}{max\{(_\alpha\leq \beta \leq 1^{max})\left[\frac{\bar{F}(\tilde{x}_\alpha^L - t)}{\tilde{x}_\alpha^L} \geq t\right]}, (_\alpha\leq \beta \leq 1^{max}\left[\frac{\bar{F}(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]$$

is increasing $x > 0$,

and hence 16

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$$\frac{min\{({}_{\alpha}\leq \beta \leq 1^{min})\left[\frac{f(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}}\geq t\right],({}_{\alpha}\leq \beta \leq 1^{min}\left[\frac{f(\tilde{x}_{\alpha}^{U}-t)}{\tilde{x}_{\alpha}^{U}}\geq t\right]}{min\{({}_{\alpha}\leq \beta \leq 1^{min})\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}}\geq t\right]},({}_{\alpha}\leq \beta \leq 1^{min}\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{U}-t)}{\tilde{x}_{\alpha}^{U}}\geq t\right]$$

$$\geq$$

$$\frac{min\{({}_{\alpha}\leq \beta \leq 1^{min})f\left(\tilde{x}_{\beta}^{L}\right),({}_{\alpha}\leq \beta \leq 1^{min}f\left(\tilde{x}_{\beta}^{U}\right)}{min\{({}_{\alpha}\leq \beta \leq 1^{min})\bar{F}\left(\tilde{y}_{\beta}^{L}\right)},({}_{\alpha}\leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}_{\beta}^{U}\right)$$

And

$$\frac{max\{({}_{\alpha}\leq \beta \leq 1^{max})\left[\frac{f(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}}\geq t\right],({}_{\alpha}\leq \beta \leq 1^{max}\left[\frac{f(\tilde{x}_{\alpha}^{U}-t)}{\tilde{x}_{\alpha}^{U}}\geq t\right]}{max\{({}_{\alpha}\leq \beta \leq 1^{max})\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}}\geq t\right]},({}_{\alpha}\leq \beta \leq 1^{max}\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{U}-t)}{\tilde{x}_{\alpha}^{U}}\geq t\right]$$

$$\geq$$

$$\frac{max\{({}_{\alpha}\leq \beta \leq 1^{max})f\left(\tilde{x}_{\beta}^{L}\right),({}_{\alpha}\leq \beta \leq 1^{max}f\left(\tilde{x}_{\beta}^{U}\right)}{max\{({}_{\alpha}\leq \beta \leq 1^{max})\bar{F}\left(\tilde{x}_{\beta}^{L}\right)},({}_{\alpha}\leq \beta \leq 1^{max}F(\tilde{x}_{\beta}^{U})$$

By defining

$$ꝁ_{\alpha}(i,x) = \frac{min\{({}_{\alpha}\leq \beta \leq 1^{min})\left[\frac{f(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}}\geq t\right],({}_{\alpha}\leq \beta \leq 1^{min}\left[\frac{f(\tilde{x}_{\alpha}^{U}-t)}{\tilde{x}_{\alpha}^{U}}\geq t\right]}{min\{({}_{\alpha}\leq \beta \leq 1^{min})\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}}\geq t\right]},({}_{\alpha}\leq \beta \leq 1^{min}\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{U}-t)}{\tilde{x}_{\alpha}^{U}}\geq t\right], i = 1$$

And

$$ꝁ_{\alpha}(i,x) = \frac{max\{({}_{\alpha}\leq \beta \leq 1^{max})\left[\frac{f(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}}\geq t\right],({}_{\alpha}\leq \beta \leq 1^{max}\left[\frac{f(\tilde{x}_{\alpha}^{U}-t)}{\tilde{x}_{\alpha}^{U}}\geq t\right]}{max\{({}_{\alpha}\leq \beta \leq 1^{max})\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}}\geq t\right]},({}_{\alpha}\leq \beta \leq 1^{max}\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{U}-t)}{\tilde{x}_{\alpha}^{U}}\geq t\right], i = 2$$

$$L_{\alpha}(x,t) = I_{0,t}(\tilde{x})\ ;\ \tilde{x},t > 0.$$

Then by definition $M_{\alpha}(i,t) = \int_0^{\infty} [ꝁ_{\alpha}(i,x).\ L_{\alpha}(x,t)]\,dx$

Thus we conclude that,

$$\frac{\left(\int_0^t min\{({}_{\alpha}\leq \beta \leq 1^{min})F(\tilde{x})_{\beta}^{L},({}_{\alpha}\leq \beta \leq 1^{min}\right)F(\tilde{x})_{\beta}^{L}\,dx}{min\{({}_{\alpha}\leq \beta \leq 1^{min})F(\tilde{t})_{\beta}^{L}},({}_{\alpha}\leq \beta \leq 1^{min}F(\tilde{t})_{\beta}^{L}$$

$$\leq \frac{\left(\int_0^t min\{(_\alpha\leq \beta \leq 1^{min})G(\tilde{y})_\beta^U, (_\alpha\leq \beta \leq 1^{min}\right)G(\tilde{y})_\beta^U\, dx}{min\{(_\alpha\leq \beta \leq 1^{min})G(\tilde{t})_\beta^U}, (_\alpha\leq \beta \leq 1^{min} G(\tilde{t})_\beta^U$$

And

$$\frac{\left(\int_0^t max\{(_\alpha\leq \beta \leq 1^{max})F(\tilde{x})_\beta^L, (_\alpha\leq \beta \leq 1^{max}\right)F(\tilde{x})_\beta^L\, dx}{max\{(_\alpha\leq \beta \leq 1^{max})F(\tilde{t})_\beta^L}, (_\alpha\leq \beta \leq 1^{max} F(\tilde{t})_\beta^L$$

$$\leq \frac{\left(\int_0^t max\{(_\alpha\leq \beta \leq 1^{max})G(\tilde{y})_\beta^U, (_\alpha\leq \beta \leq 1^{max}\right)G(\tilde{y})_\beta^U\, dx}{max\{(_\alpha\leq \beta \leq 1^{max})G(\tilde{t})_\beta^U}, (_\alpha\leq \beta \leq 1^{max} G(\tilde{t})_\beta^U$$

Is decreasing in $x > 0$. Now using previous lemma proof is complete.

The other parts can be proved in a similar way.

5.4 THEOREM:

Let X and Y are two fuzzy random variables have a common fuzzy mean past life

F($\tilde{x}$) and G($\tilde{y}$) respectively. Then, (a)If,

$$\frac{min\{(_\alpha\leq \beta \leq 1^{min})\left[\frac{f(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right], (_\alpha\leq \beta \leq 1^{min}\left[\frac{f(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]}{min\{(_\alpha\leq \beta \leq 1^{min})\left[\frac{\bar{F}(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]}, (_\alpha\leq \beta \leq 1^{min}\left[\frac{\bar{F}(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]$$

$$\geq$$

$$\frac{min\{(_\alpha\leq \beta \leq 1^{min})f\left(\tilde{x}_\beta^L\right), (_\alpha\leq \beta \leq 1^{min} f\left(\tilde{x}_\beta^U\right)}{min\{(_\alpha\leq \beta \leq 1^{min})\bar{F}\left(\tilde{y}_\beta^L\right)}, (_\alpha\leq \beta \leq 1^{min} \bar{F}\left(\tilde{x}_\beta^U\right)$$

And

$$\frac{max\{(_\alpha\leq \beta \leq 1^{max})\left[\frac{f(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right], (_\alpha\leq \beta \leq 1^{max}\left[\frac{f(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]}{max\{(_\alpha\leq \beta \leq 1^{max})\left[\frac{\bar{F}(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]}, (_\alpha\leq \beta \leq 1^{max}\left[\frac{\bar{F}(\tilde{x}_\alpha^U - t)}{\tilde{x}_\alpha^U} \geq t\right]$$

$$\geq$$

$$\frac{max\{(_\alpha\leq \beta \leq 1^{max})f\left(\tilde{x}_\beta^L\right), (_\alpha\leq \beta \leq 1^{max} f\left(\tilde{x}_\beta^U\right)}{max\{(_\alpha\leq \beta \leq 1^{max})\bar{F}\left(\tilde{x}_\beta^L\right)}, (_\alpha\leq \beta \leq 1^{max} F\left(\tilde{x}_\beta^U\right)$$

Is increasing in t , then X $\leq_{FRH1}$ Y if and only if X $\leq_{FMIT4}$ Y

(b) If,

$$\frac{min\{({}_{\alpha}\leq \beta \leq 1^{min})\left[\frac{f(\tilde{x}^{L}_{\alpha}-t)}{\tilde{x}^{L}_{\alpha}}\geq t\right],({}_{\alpha}\leq \beta \leq 1^{min}\left[\frac{f(\tilde{x}^{L}_{\alpha}-t)}{\tilde{x}^{L}_{\alpha}}\geq t\right]}{min\{({}_{\alpha}\leq \beta \leq 1^{min})\left[\frac{\bar{F}(\tilde{x}^{U}_{\alpha}-t)}{\tilde{x}^{U}_{\alpha}}\geq t\right]},({}_{\alpha}\leq \beta \leq 1^{min}\left[\frac{\bar{F}(\tilde{x}^{U}_{\alpha}-t)}{\tilde{x}^{U}_{\alpha}}\geq t\right]$$

$$\geq$$

$$\frac{min\{({}_{\alpha}\leq \beta \leq 1^{min})f\left(\tilde{x}^{L}_{\beta}\right),({}_{\alpha}\leq \beta \leq 1^{min}f\left(\tilde{x}^{L}_{\beta}\right)}{min\{({}_{\alpha}\leq \beta \leq 1^{min})\bar{F}\left(\tilde{y}^{L}_{\beta}\right)},({}_{\alpha}\leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}^{L}_{\beta}\right)$$

And

$$\frac{max\{({}_{\alpha}\leq \beta \leq 1^{max})\left[\frac{f(\tilde{x}^{L}_{\alpha}-t)}{\tilde{x}^{L}_{\alpha}}\geq t\right],({}_{\alpha}\leq \beta \leq 1^{max}\left[\frac{f(\tilde{x}^{L}_{\alpha}-t)}{\tilde{x}^{L}_{\alpha}}\geq t\right]}{max\{({}_{\alpha}\leq \beta \leq 1^{max})\left[\frac{\bar{F}(\tilde{x}^{L}_{\alpha}-t)}{\tilde{x}^{L}_{\alpha}}\geq t\right]},({}_{\alpha}\leq \beta \leq 1^{max}\left[\frac{\bar{F}(\tilde{x}^{L}_{\alpha}-t)}{\tilde{x}^{L}_{\alpha}}\geq t\right]$$

$$\geq$$

$$\frac{max\{({}_{\alpha}\leq \beta \leq 1^{max})f\left(\tilde{x}^{L}_{\beta}\right),({}_{\alpha}\leq \beta \leq 1^{max}f\left(\tilde{x}^{L}_{\beta}\right)}{max\{({}_{\alpha}\leq \beta \leq 1^{max})\bar{F}\left(\tilde{x}^{L}_{\beta}\right)},({}_{\alpha}\leq \beta \leq 1^{max}\bar{F}(\tilde{x}^{L}_{\beta})$$

Is increasing in t , then X $\leq_{FRH2}$ Y if and only if X $\leq_{FMIT2}$ Y.

(C) If,

$$\frac{min\{({}_{\alpha}\leq \beta \leq 1^{min})\left[\frac{f(\tilde{x}^{U}_{\alpha}-t)}{\tilde{x}^{U}_{\alpha}}\geq t\right],({}_{\alpha}\leq \beta \leq 1^{min}\left[\frac{f(\tilde{x}^{U}_{\alpha}-t)}{\tilde{x}^{U}_{\alpha}}\geq t\right]}{min\{({}_{\alpha}\leq \beta \leq 1^{min})\left[\frac{\bar{F}(\tilde{x}^{U}_{\alpha}-t)}{\tilde{x}^{U}_{\alpha}}\geq t\right]},({}_{\alpha}\leq \beta \leq 1^{min}\left[\frac{\bar{F}(\tilde{x}^{U}_{\alpha}-t)}{\tilde{x}^{U}_{\alpha}}\geq t\right]$$

$$\geq$$

$$\frac{min\{({}_{\alpha}\leq \beta \leq 1^{min})f\left(\tilde{x}^{U}_{\beta}\right),({}_{\alpha}\leq \beta \leq 1^{min}f\left(\tilde{x}^{U}_{\beta}\right)}{min\{({}_{\alpha}\leq \beta \leq 1^{min})\bar{F}\left(\tilde{y}^{U}_{\beta}\right)},({}_{\alpha}\leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}^{U}_{\beta}\right)$$

And

$$\frac{max\{(_{\alpha}\leq \beta \leq 1^{max})\left[\frac{f(\tilde{x}_{\alpha}^{U}-t)}{\tilde{x}_{\alpha}^{U}} \geq t\right], (_{\alpha}\leq \beta \leq 1^{max}\left[\frac{f(\tilde{x}_{\alpha}^{U}-t)}{\tilde{x}_{\alpha}^{U}} \geq t\right]}{max\{(_{\alpha}\leq \beta \leq 1^{max})\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{U}-t)}{\tilde{x}_{\alpha}^{U}} \geq t\right]}, (_{\alpha}\leq \beta \leq 1^{max}\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{U}-t)}{\tilde{x}_{\alpha}^{U}} \geq$$

$$\geq$$

$$\frac{min\{(_{\alpha}\leq \beta \leq 1^{min})f\left(\tilde{x}_{\beta}^{U}\right), (_{\alpha}\leq \beta \leq 1^{min}f\left(\tilde{x}_{\beta}^{U}\right)}{min\{(_{\alpha}\leq \beta \leq 1^{min})\bar{F}\left(\tilde{y}_{\beta}^{U}\right)}, (_{\alpha}\leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}_{\beta}^{U}\right)$$

Is increasing in t , then X $\leq_{FRH3}$ Y if and only if X $\leq_{FMIT3}$ Y.

(d) If,

$$\frac{min\{(_{\alpha}\leq \beta \leq 1^{min})\left[\frac{f(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}} \geq t\right], (_{\alpha}\leq \beta \leq 1^{min}\left[\frac{f(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}} \geq t\right]}{min\{(_{\alpha}\leq \beta \leq 1^{min})\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}} \geq t\right]}, (_{\alpha}\leq \beta \leq 1^{min}\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}} \geq t\right]$$

$$\geq$$

$$\frac{min\{(_{\alpha}\leq \beta \leq 1^{min})f\left(\tilde{x}_{\beta}^{U}\right), (_{\alpha}\leq \beta \leq 1^{min}f\left(\tilde{x}_{\beta}^{U}\right)}{min\{(_{\alpha}\leq \beta \leq 1^{min})\bar{F}\left(\tilde{y}_{\beta}^{U}\right)}, (_{\alpha}\leq \beta \leq 1^{min}\bar{F}\left(\tilde{x}_{\beta}^{U}\right)$$

And

$$\frac{max\{(_{\alpha}\leq \beta \leq 1^{max})\left[\frac{f(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}} \geq t\right], (_{\alpha}\leq \beta \leq 1^{max}\left[\frac{f(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}} \geq t\right]}{max\{(_{\alpha}\leq \beta \leq 1^{max})\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}} \geq t\right]}, (_{\alpha}\leq \beta \leq 1^{max}\left[\frac{\bar{F}(\tilde{x}_{\alpha}^{L}-t)}{\tilde{x}_{\alpha}^{L}} \geq t\right]$$

$$\geq$$

$$\frac{max\{(_{\alpha}\leq \beta \leq 1^{max})f\left(\tilde{x}_{\beta}^{U}\right), (_{\alpha}\leq \beta \leq 1^{max}f\left(\tilde{x}_{\beta}^{U}\right)}{max\{(_{\alpha}\leq \beta \leq 1^{max})\bar{F}\left(\tilde{x}_{\beta}^{U}\right)}, (_{\alpha}\leq \beta \leq 1^{max}F\left(\tilde{x}_{\beta}^{U}\right)$$

Is increasing in t > 0, then X $\leq_{FRH4}$ Y if and only if X $\leq_{FMIT1}$ Y.

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